A METHODOLOGY FOR SELECTING AMONG WATER QUALITY ALTERNATIVES

By

GILBERT S. NICOLSON. JR.

A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF
THE UNIVERSITY OF FLORIDA
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA
1969

ACKNOWLEDGMENTS

The author wishes to acknowledge the support furnished by the U. S. Army Corps of Engineers. This dissertation was prepared under the Corps' Graduate Fellowship Program, and any reproduction, in whole or in part, for the purpose of the United States Government is authorized.

The author is very grateful to his chairman, Dr. E. E. Pyatt, who was a source of encouragement and inspiration throughout this study. The help and guidance of Dr. R. L. Patterson, Dr. W. C. Huber, and Dr. N. G. Keig during the course of this work were of inestimable value. The suggestions of Dr. J. P. Heaney for improving the manuscript were appreciated. The author also acknowledges the help of Dr. J. C. Schaake and Dr. D. H. Moreau, whose ideas and insights provided the foundation for this study.

The author is grateful to the personnel of the Corps of Engineers: Mr. Herb Rogers of the South Atlantic Division Office, Mr. Angelo Tabita of the Jacksonville District Office, and Messrs. Frank Posey and Joe DeWitt of the Savannah District Office. The assistance afforded the author during the Fellowship Program by the personnel of the Project Planning Branch of the Jacksonville District Office especially is appreciated.

This dissertation never could have been completed without the encouragement and understanding of the author's wife. Her faith has been an inspiration throughout the author's graduate career and for this he is eternally grateful.

The author acknowledges the aid of Mrs. Linda Smith in the typing and preparation of the manuscript. The typing of Mrs. Jean Weidner also is greatly appreciated. To Mr. Bob Jandrucko special thanks are given for his preparation of the drawings.

The help of the author's colleagues was greatly appreciated. Their many suggestions and encouragement added to the successful completion of this study.

The author acknowledges the financial support of a Public Health Service Traineeship during his early graduate studies. Computer time was furnished under FWPCA Grant No. WP01050 and by the University of Florida Computing Center.

TABLE OF CONTENTS

		Page
ACKNOWLEDGMENTS		ii
LIST OF TABLES		vi
LIST OF FIGURES		vii
ABSTRACT		viii
CHAPTER		
I.	INTRODUCTION AND OBJECTIVES	1
	Introduction	1
	Historical Background	1
	The Parameter of Major Interest	3
	Water Quality Alternatives	5
	Objectives	7
II.	RESERVOIR EFFECTS ON STREAM QUALITY	8
	Effects on Temperature	8
	Heat Budget	10
	Internal Transfer Mechanisms	14
	The Heat Transfer Equation	16
	Thermal Stratification of Lakes	18
	Density Currents	24
	Reservoir Prediction Models	28
	Reservoir Temperature Model Used in This Study	34
	Dissolved Oxygen	40
	Improving Release Water Quality	46
	Method Used in This Study	48
III.	STREAM TEMPERATURE AND DISSOLVED OXYGEN MODELS	51
	Stream Temperature	51
	Dissolved Oxygen	56
	Determination of Allowable Waste Loading	69
IV.	THE TRADEOFF BETWEEN STREAM TEMPERATURE AND STREAM FLOW	72
	The Effects of Temperature on Waste Assimilative Capacity	72
	The Effects of Stream Flow on Waste Assimilative Capacity	75
	The Stream Flow Stream Temperature Tradeoff	76

Page

V. DEVELOPMENT OF METHODOLOGY 79

Description of the Problem 79
Water Quality Optimization Models 82
Dynamic Programming 88

VI. DATA REQUIREMENTS 110

VII. TEST CASE 114

Description of River Basin 114
Model Verification 121
Waste Dischrage Model 124
Discharge Rates 131
Alternatives Examined and Results 132
Summary of Test Case 139

VIII. SUMMARY 140

APPENDICES

A. PROBABILISTIC DEFICIT FUNCTION 145

B. PROGRAM LISTING 150

C. GLOSSARY OF PROGRAMMING TERMS 176

REFERENCES 180

BIOGRAPHICAL SKETCH 185

LIST OF TABLES

Table		Page
1.	Proportionality Coefficients and Intercepts for the Temperature, Vapor Pressure Approximation	38
2.	Sensivity Analysis	67
3.	Cost Matrix for Dynamic Programming Example	91
4.	Evaluation of Equation (78) for N Equal Two in Example	93
5.	Evaluation of Equation (78) for N Equal Three in Example	94
6.	Evaluation of Equation (78) for N Equal Four in Example	95
7.	Reservoir Dissolved Oxygen Constants and Epilimnion Depth Variation	117
8.	River Hydraulic Data	119
9.	Distribution of Ungaged Runoff	120
10.	Reservoir Release Temperatures	123
11.	Base Flow	126
12.	Linear Regression Results	130
13.	Data Listing for Trial Runs	133
14.	Minimum Total Costs for Various Alternatives	134
15.	Optimum Decisions for Run A	135
16.	Optimum Decisions for Run C	136
17.	Optimum Decisions for Runs F and G	137
18.	Results of Runs for Different k_1 Values	138

LIST OF FIGURES

Figure		Page
1.	Stream Temperature vs. Time	9
2.	Coordinate System	17
3.	Annual Temperature Cycle for Lakes in Temperate Zones	19
4.	Eddy Diffusivity Coefficient and Temperature vs. Depth	22
5.	Temperature Profiles as Predicted by Dake and Harleman	25
6.	Inflow and Outflow Velocity Profiles After Huber and Harleman	30
7.	Assumed Velocity Profiles	36
8a.	Dissolved Oxygen Variation with Time and Depth	43
8b.	Temperature Variation with Time and Depth	44
9.	Matrix for Representing Dissolved Oxygen Variation Within the Reservoir	49
10.	Dissolved Oxygen Sag Curve	59
11.	Hypothetical Curves for Waste Assimilative Capacity vs. Release Temperature and Flow	77
12.	River-Reservoir System	80
13.	Dynamic Programming Flow Chart	98
14.	Savannah River Basin	116
15.	Reservoir Surface Temperatures	122
16.	Downstream Temperature Profiles for August 1957	125
17.	Waste Assimilative Capacity vs. Temperature and Flow	127
18.	Waste Assimilative Capacity vs. Equilibrium Temperature and Dissolved Oxygen	128

Abstract of Dissertation Presented to the Graduate Council in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy.

A METHODOLOGY FOR SELECTING AMONG WATER QUALITY ALTERNATIVES

By

Gilbert S. Nicolson, Jr.

June, 1969

Chairman: E. E. Pyatt
Major Department: Environmental Engineering

The Water Quality Act of 1965 requires each state to establish stream quality standards. The major cause of violation of the standards is municipal and industrial waste discharges. To maintain the standards most state enforcement agencies require a high degree of waste treatment. Such requirements entail a large expenditure of private and public funds for maintaining the standards during low-flow periods. These funds could be used more efficiently if provisions were made for the examination of alternative schemes.

In the case of a municipality downstream from a reservoir, several alternatives exist both at the reservoir and the waste treatment plant. One of the alternatives at the treatment plant is an auxiliary unit which could be operated during low-flow periods. Alternatives at the reservoir site include multi-level outlet structures, reaeration of releases, and reservoir mixing. This dissertation presents a methodology for evaluation of these alternatives. For each alternative considered, optimal decisions are made which minimize the total waste treatment costs of the auxiliary unit. Comparison of the costs of providing the alternative enable the selection of the least-cost alternative.

The concept of a water quality tradeoff is introduced. The tradeoff exists in reservoirs which thermally stratify during the summer months. Stratification permits selective withdrawal from the reservoir and a substitution of water at one temperature and flow rate for water at another temperature and flow rate while still maintaining water quality standards.

The methodology utilizes a two-dimensional dynamic programming algorithm for the optimization technique. Several water quality models are needed to determine the reservoir release temperatures, the downstream temperature profiles, and the dissolved oxygen response to waste loads. Detailed analyses of the various models are presented. A stochastic dissolved oxygen formulation is presented which permits the determination of waste loads that satisfy the stream standard with a given probability.

A test case is presented which illustrates the use of the methodology. Included in the test case is the verification of the temperature models used in the study. The existence of a water quality tradeoff is demonstrated and the costs for several alternatives are determined. The methodology is shown to be sensitive to the deoxygenation coefficient, and it is concluded that a good estimate of the coefficient is needed for obtaining reliable cost comparison.

I. INTRODUCTION AND OBJECTIVES

Introduction

During the rapid technological expansion of the twentieth century, man's capacity to produce waste has caused deterioration of his environment. The vast quantities of waste and the sometimes acute health hazards and unpleasant esthetic conditions have produced a public awareness of the need to protect the environment, not only for the present generation but also for posterity.

One of the most important natural resources that must be preserved for man's existence is water, which is used for recreation, navigation, water supply, power production, maintenance of fish and wildlife, irrigation, and assimilation of domestic and industrial waste. It is this last use that is threatening the majority of the other uses and that recent federal legislation is designed to control.

Historical Background

Federal interest in water was initially for the purpose of navigation and flood control. Public Law 845 enacted by the Eightieth Congress was the first federal legislation intended for pollution abatement.[1] This law provided for monies to be spent for aiding in design and construction of pollution abatement facilities. Since 1948, several laws have been enacted, each with the general purpose of

providing more funds for abatement facilities. In 1963, Congress directed that all new multipurpose reservoir projects should consider additional storage for releases during periods of low flow.[2] This was the first step toward water quality control in reservoir projects. However, Congress specified that such releases should not be made in lieu of waste treatment.

With the Water Quality Act of 1965, Congress transferred the Division of Water Supply and Pollution Control within the U. S. Public Health Service to create the Federal Water Pollution Control Administration (FWPCA) in the Department of the Interior. The Act required that each state set water quality standards subject to approval by the FWPCA.[3] The water quality standards should be designed to "enhance the quality and value of our national resources...."[3] In the "Guidelines for Establishing Water Quality Standards for Interstate Waters"[4] the FWPCA stated that "no standard will be approved which allows any wastes amenable to treatment or control to be discharged into any interstate water without treatment or control regardless of the water quality criteria and water uses adopted."

The Water Quality Act of 1965, therefore, provides for the establishment of quality standards for the preservation and maintenance of one sector of man's environment. The maintenance of water quality standards implies a vast outlay of public funds since all water-borne waste must be subject to treatment and control.[5] The Water Resource Engineer must undertake the responsibility of providing the best engineering plans for the expenditure of these funds.

Since water quality standards are established for an entire basin, plans for meeting the standards should apply to the basin or to

independent parts of the basin. Water resource projects usually have been designed according to the theory of efficient allocation of resources. In the past, the efficiency criterion provided for the maximization of net benefits. The engineer would list the purposes for which the project was to be built. For each purpose he would perform a benefit-cost analysis. The purpose would be included in the project if the benefits accruing to this purpose exceeded the cost of providing it. The scale of each purpose would be increased until the benefits equaled the costs; therefore, net benefits would be maximized.

With the establishment of water quality standards, the theory of efficient allocation still can be satisfied. However, the constraint is modified from requiring that benefits exceed costs to satisfying the water quality standards. By changing the constraint, it is possible to alter the efficiency criterion to selecting the least-cost method which achieves the water quality objectives. This criterion does not require that the standards be established under the principle of maximizing net benefits. One can view water quality control under the stream standards as a problem of maximizing net benefits. If water quality control is assumed a benefit, the least-cost alternative which maintains the standards will by necessity maximize net benefits. In either case, the major difficulty is determining the costs of the alternatives.

The Parameter of Major Interest

As mentioned earlier, it was the deterioration of one of man's most vital resources coupled with lethargic state programs which precipitated

federal legislation. The stream standards are established for the purpose of preventing further pollution and improving water quality in badly polluted streams. Therefore, the parameters of primary concern should be the water quality parameter which is sensitive to pollution and for which suitable mathematical expressions exist. These expressions should trace the response of the parameter to waste loads and provide a prediction of the response as a function of the waste loading. The water quality parameter frequently used for pollution investigations is dissolved oxygen. There are many reasons for the selection of dissolved oxygen: first, it can be accurately measured; second, the aquatic biota and bacteria which assimilate the organic content of the waste require dissolved oxygen to maintain their metabolic processes; third, the level of dissolved oxygen can be a determining factor as to the species of biota that can survive in the stream;[6] fourth, the absence of dissolved oxygen is readily associated with severe pollution. For these reasons, dissolved oxygen will be the primary water quality parameter considered in this study. Since dissolved oxygen is an important resource with respect to the stream's capacity to assimilate organic or inorganic waste, any action taken to increase the concentration of dissolved oxygen or the total oxygen resources would also increase the stream's waste assimilative capacity.

A stream's waste assimilative capacity is affected by the temperature of the stream. The stream's dissolved oxygen saturation level is markedly affected by temperature, and the rate at which organisms decompose the waste also is influenced by temperature. Placing a reservoir on a stream alters the stream temperature regime below the reservoir. Since

the reservoir is a point of control in the basin, it is possible to change the stream's assimilative capacity simply by controlling the temperature of the release water. (The effects of reservoirs on stream temperature and methods for controlling the releases will be detailed in a later chapter.) The major concern of the present study deals with the importance of temperature and methods of improving waste assimilative capacity through temperature control. Other methods of water quality control are reservoir mixing, turbine reaeration to increase the dissolved oxygen in reservoir releases, and in-stream reaeration.

Water Quality Alternatives

There are several methods employed either to increase the allowable waste loading or to alleviate the detrimental effects of pollution. The method quite often employed is reducing the waste discharge into the stream, which can be accomplished by installing waste treatment facilities or by altering industrial processes to reduce waste. In the case of waste treatment facilities, considerable capital investment by private and public institutions is necessary to achieve high levels of waste treatment. These monies should not be spent without some consideration to alternative means of meeting water quality standards.

A second method of alleviating pollution problems is to provide adequate dilution of wastes. In most cases, water quality deterioration is most severe during the summer and fall months when the natural stream flow is lowest. Storing water in reservoirs for release during low-flow periods has become a method of water quality control. Bramhall and Mills[7] criticized the use of low-flow augmentation as an

alternative method of meeting stream standards when compared to high levels of waste treatment. However, in their analysis the authors considered reservoir storage only for use as low-flow augmentation, which would introduce a bias in favor of waste treatment. Young[8] pointed out the compatibility of flood control storage and low-flow augmentation. Flow augmentation seems to be technically feasible as a means of maintaining water quality objectives and should be considered as an alternative to waste treatment.

A third method of water quality control is waste storage during times when stream conditions are such that the waste discharge would cause violation of stream standards. Loucks[9] illustrated the use of waste storage in the establishment of probabilistic stream standards. Loucks did not consider any waste treatment in the storage facility, but Moreau[10] presented an analysis for the pulp and paper industry of a treatment system composed of steady-state and stochastic units. The effluent from the steady-state unit always received a constant degree of treatment. When the receiving stream was unable to accept the constant effluent, a portion of the waste went to the stochastic unit where it received sufficient treatment so that water quality standards were not violated. Therefore, an alternative to a high degree of treatment the year around would be the storing or treating part of the waste in a variable waste treatment unit during low flow periods.

Objectives

In planning to meet water quality standards, all alternative schemes and combinations should be considered in attempting to find the least-cost alternative. A methodology or framework is needed for comparing water quality alternatives in order to make the best decision. The objective of this disseration is to develop a methodology for selecting from among water quality control alternatives for a reservoir river system. The methodology will employ mathematical expressions for tracing the temperature and dissolved oxygen profiles, an inventory system for reservoir operation and a dynamic programming technique for selecting optimal reservoir releases.

In Chapter II the effects of a reservoir on temperature and dissolved oxygen along with the models used to predict them will be discussed. Chapter III details the stream models needed to calculate downstream temperature and dissolved oxygen. A discussion of the tradeoff between reservoir release temperatures and flows will be presented in Chapter IV, and the development of the methodology to make use of this tradeoff will be formulated in Chapter V. Data requirements for the proposed methodology will be discussed in Chapter VI. Chapter VII will consist of the test case illustrating the use of the methodology and Chapter VIII will contain the summary and conclusions. Flow charts of programs and a glossary of terms will be placed in the Appendices.

II. RESERVOIR EFFECTS ON STREAM QUALITY

To develop alternative plans for meeting water quality standards in a reservoir-river system, one must first understand the changes imposed on the river with the introduction of a reservoir. This chapter presents a discussion of the two quality parameters of particular interest in this study: temperature and dissolved oxygen. The discussion will include reservoir effects, methods used to predict or describe the effects, and the model selected for this study.

Effects on Temperature

Ward[11] has studied stream temperatures in regulated and unregulated rivers and has proposed an expression for representing the temperature as a function of time:

$$T = a\,[\sin\,(bt + c)] + \bar{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where T is the stream temperature in degrees Fahrenheit; a is the amplitude in degrees Fahrenheit;

$$b = \frac{360^\circ}{365 \text{ days}} = .987 \text{ degrees per day;}$$

t is the number of days since October 1; c is the phase coefficient in degrees; and $\bar{T}$ is the average annual temperature in degrees Fahrenheit. The coefficients a, b and c are determined from a least-squares analysis of the stream temperature data. Ward calculated these coefficients for several rivers in Arkansas. Figure 1 illustrates the effect of

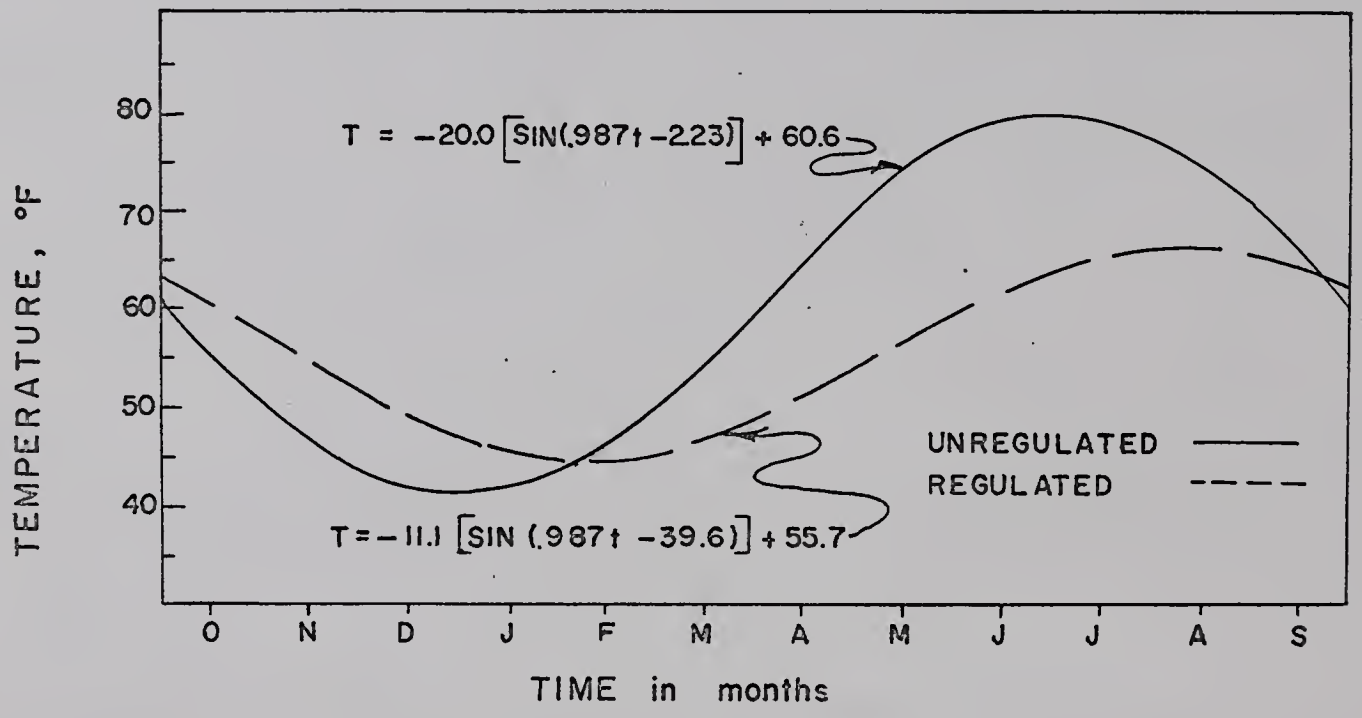


FIGURE 1. STREAM TEMPERATURE vs. TIME

placing the Bull Shoals Reservoir on the White River: the average temperature $\bar{T}$ was lower; the annual variation 2a decreased; and the phase coefficient $|c|$ increased, causing approximately a forty-day lag in the sine curve. This represents the change in the time variation of stream temperature for one specific reservoir, but, depending on reservoir operation and outlet elevations, this is a typical before-and-after example. Churchill[12, 13] has studied several cases of the temperature change caused by reservoirs built by the Tennessee Valley Authority. In each case, the effects were similar to those in Figure 1.

Before the models used to describe the changes in reservoir temperatures can be discussed, it is necessary to understand the relationship between the heat budget of a body of water and the mechanisms of heat transfer in the water and the role each plays during the annual cycle of reservoir temperatures. Most methods for predicting water temperature depend upon the determination of the water's heat budget, which is a summation of the available sources and sinks of heat acting on the water. Calculations for the heat budget are made to obtain the net rate of heat transfer at the surface. The development of the equations for the heat budget will follow the work of Edinger and Geyer.[14]

Heat Budget

There are seven major mechanisms of heat transfer at the water surface. These are incoming short-wave solar radiation, H_s; incoming long-wave radiation H_a; reflected short-wave and long-wave radiation, H_{sr} and H_{ar}; long-wave back radiation, H_{br}; heat conduction, H_c; and heat loss by evaporation, H_e. Following is a brief discussion of each of these terms and their

relative magnitudes. (For a more detailed discussion, the reader is referred to Edinger and Geyer[14] or to the Report of the 1960-61 Advanced Seminar of the Johns Hopkins University, Department of Sanitary Engineering and Water Resources.[15])

Absorbed Radiation

Short-wave solar radiation reaching any point on the earth depends upon the latitude, cloud cover, sun's altitude, and time of year. Empirical formulas are sometimes used for estimating short-wave radiation, but it is measured more easily than it is computed. The U.S. Weather Bureau records short-wave radiation and reports the measurements in Climatological Data -- National Summary. Monthly values for solar radiation in the northern latitudes range from 400 - 2800 Btu per square foot per day. The solar radiation incident on a body of water is absorbed in the first ten to twenty feet, depending upon the optical properties of the water.[16]

Long-wave atmospheric radiation is a function of cloud cover, air temperature and air vapor pressure. Raphael presents the following formula for long-wave radiation:[17]

$$H_a = \beta_o \sigma (T_a + 460)^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where β_o is an atmospheric radiation factor depending on cloud cover and air vapor pressure; σ is the Stefan-Boltzman constant (4.15×10^{-8} Btu per square foot per degrees Rankine per day); and T_a is the air temperature in degrees Fahrenheit. Long-wave radiation ranges from 2400 - 3200 Btu per square foot per day.

Reflected long and short-wave radiation are calculated as fractions of incident long and short-wave radiation. Reflected radiations can

range from 100 - 300 Btu per square foot per day.

$$H_{ar} \doteq .03\, H_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$H_{sr} \doteq .05\, H_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Edinger and Geyer combine H_s, H_a, H_{ar} and H_{sr} into one term called the absorbed radiation, H_R.

$$H_R = H_s + H_a - H_{sr} - H_{ar} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

The absorbed radiation is independent of the water temperature and is, therefore, conveniently calculated from meteorological data. Absorbed radiation ranges from 2500 - 5700 Btu per square foot per day.

Heat Losses

Long-wave back radiation is emitted from the body of water and depends on the temperature at the water surface. The equation for back radiation is:

$$H_{br} = \sigma\ \gamma_w\ (T_s + 460)^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where σ is the Stefan-Boltzman constant; γ_w is the emissivity of water (0.97); and T_s is the surface water temperature in degrees Fahrenheit. Back radiation is a substantial heat sink and ranges from 2400 - 3600 Btu per square foot per day.

Another large heat sink is evaporation, which is a function of wind speed and the vapor pressure gradient between the air and the water surface. Several empirical formulas are used for calculation of heat loss due to the latent heat of vaporization. The general form of the equation is:

$$H_e = (a + bW)\ (e_s - e_a) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

where a and b are constants depending upon the evaporation formula

employed; W is wind speed in miles per hour, mph; e_s is the saturation vapor pressure of water in millimeters of mercury, mm Hg, determined from the water surface temperature, T_s; and e_a is the air-vapor pressure in mm Hg. For the Lake Hefner Formula, a and b are 0 and 11.4, respectively; while in the Meyer Equation, for monthly evaporation, they are 73 and 7.3,respectively.[14] Evaporative heat loss ranges from 2000 - 8000 Btu per square foot per day.

Heat conduction is positive or negative depending upon whether the difference between the air temperature and the water temperature is positive or negative. The Bowen ratio is used to relate heat conduction to evaporative heat loss.[17]

$$B = \frac{H_c}{H_e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

B is the Bowen ratio and is determined from the equation

$$B = \frac{C\,(T_s - T_a)}{(e_s - e_a)} \cdot \frac{P}{760} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

where T_s, T_a, e_s, and e_a are as defined in preceding equations; P is the atmospheric pressure in mm Hg; and C is a coefficient determined from experiments to be approximately 0.26. Substituting Equations (9) and (7) into Equation (8) and assuming P is 760 mm Hg yields:

$$H_c = 0.26\,(a + bW)\,(T_s - T_a) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

Heat conduction can range from a minus 320 to a positive 400 Btu per square foot per day.

Net Heat Transfer

The net rate of heat transfer, ΔH, is the algebraic sum of the absorbed radiation, back radiation, evaporative heat loss, and conductive heat losses. Assuming conductive heat transfer as positive

when the air temperature is greater than the water temperature, the equation for net heat transfer is:

$$\Delta H = H_R - (H_{br} + H_e + H_c) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

The following discussion of Equation (11) is abstracted from Edinger and Geyer. When the absorbed radiation, H_R, is greater than the rate at which heat is lost by a body of water through evaporation, conduction, and back radiation, the net rate at which heat enters through the water surface is positive ($\Delta H > 0$) and heat is added to the water. Heat leaves the water ($\Delta H > 0$) when absorbed radiation is less than the rate at which heat is lost. When the net rate of heat transfer is zero ($\Delta H = 0$), a special condition arises, which leads to the definition of the equilibrium temperature. (The role of the equilibrium temperature in reservoir temperature prediction will be discussed later in this chapter.) Substituting Equations (6), (7) and (10) into Equation (11) yields:

$$\Delta H = H_R - \sigma \gamma \ (T_s + 460)^4 - (a + bW)(e_s - e_a) - .26\,(a + bW)(T_s - T_a) \quad \ldots\ldots\ldots\ldots \quad (12)$$

Equation (12) is referred to as the "basic" heat transfer equation because it can be derived without any assumptions as to the type of water being studied or specification of an evaporation formula.[14] Equation (12) cannot be used to predict water surface temperatures unless some other method is used for obtaining ΔH. However, Equation (12) will be used to derive the predictive equation used in this study.

Internal Transfer Mechanisms

The discussion of the heat budget was limited to the heat transfer mechanisms at the water surface. The heat transfer mechanisms which

distribute the heat downward into the reservoir are the core of the predictive reservoir temperature models. There are two modes of heat transfer, other than absorption, in a body of water which distribute the heat vertically.

One mode of heat transfer in a body of water is advection. Advective heat transfer is the result of heat transferred in a flowing fluid by macroscopic particles. The source of advected heat in reservoirs is inflow, either surface or underground, outflows and rainfall. In most reservoir studies, advected heat from inflow and outflow is an important heat transfer mechanism.

The second mode of heat transfer is diffusion. Diffusion in water can be either molecular diffusion or eddy diffusion. The heat flux per unit area due to molecular diffusion is given by the product of the temperature gradient and the molecular diffusivity coefficient α. The diffusivity coefficient in water is very small in comparison to eddy diffusivity ($\alpha = .00144$ cm^2/sec) and is considered independent of water temperature.

Heat flux by eddy diffusion is the result of eddy diffusivity caused by wind-induced currents, convectional currents resulting from surface waters cooling and sinking to lower depths, inflow currents from tributaries to the reservoir, or outflow currents. Wunderlich and Elder[19] present a detailed discussion of what they term turbulent diffusion mechanisms and their importance in reservoirs. They indicate that convective currents are the most important source of eddy diffusivity in the surface layer of reservoirs and that inflow and withdrawal currents are important in the lower layers. The heat flux per unit area by eddy diffusion is the product of the thermal

gradient and the eddy diffusivity coefficient (E). The eddy diffusivity coefficient is not constant and varies from point to point in the reservoir and from time to time. The eddy diffusivity coefficient is large in comparison to the molecular diffusivity coefficient (E ranges from .01 to 10 cm^2/sec).

The Heat Transfer Equation

Basic mathematical expressions representing the modes of heat transfer can be combined into a differential equation for the rate of heat exchange within the water. The equation for heat transfer in a fluid will be derived making use of the principle of conservation of heat. This principle states that the rate of change of heat within the body (stored heat) is equal to the difference between the rate of heat inflow and rate of heat outflows by the mass transport mechanisms plus the rate of external heat added or lost. In equation form this is:

[rate of change of stored heat] = [rate of heat inflow by mass transport] - [rate of heat outflow by mass transport] + [rate of external heat added] ..... (13)

The three-dimensional coordinate system and unit cube shown in Figure 2 are used in the derivation. The heat content of any body is given as

$$\rho c_p T$$

where ρ is the density of the substance in pounds per cubic foot; c_p is the specific heat of the substance in Btu per pound per degree Fahrenheit; and T is the temperature of the substance in degrees Fahrenheit. For this derivation the substance will be water, in

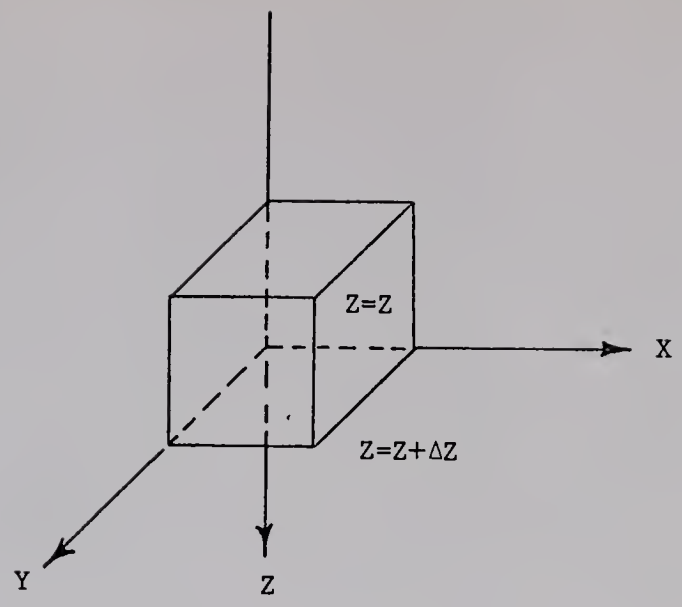


FIGURE 2. COORDINATE SYSTEM

which the density and specific heat can be considered constants. The rate of change of the heat content of the cube is

$$\rho c_p \frac{\partial T}{\partial t}$$

where t is time.

The heat advected across the x-y plane in the positive z-direction is:

$$\rho c_p v_x T\Big|_{z = z+\Delta z} - \rho c_p v_x T\Big|_{z = z} \equiv \rho c_p v_x \frac{\partial T}{\partial x} \Delta z$$

where v_x is velocity in the x-direction in feet per day.

The heat diffused across the x-y plane in the positive z direction by molecular and eddy diffusion is:

$$\rho c_p (E_x + \alpha) \frac{\partial T}{\partial x}\Big|_{z = z+\Delta z} - \rho c_p (E_x + \alpha) \frac{\partial T}{\partial x}\Big|_{z = z} \equiv \rho c_p \frac{\partial}{\partial x} [(E_x+\alpha)\frac{\partial T}{\partial x}] \Delta z$$

where E_x is the eddy diffusivity coefficient in the x-direction in square feet per day.

Let the external heat term be represented by S in Btu per cubic feet per day. Taking the sum of the advected and diffused terms in the x, y and z-directions, substituting the appropriate terms into Equation (13), and dividing by c_p, the final expression becomes:

$$\frac{\partial T}{\partial t} = [\frac{\partial}{\partial x}(E_x + \alpha)\frac{\partial T}{\partial x}] + [\frac{\partial}{\partial y}(E_y + \alpha)\frac{\partial T}{\partial y}] + [(\frac{\partial}{\partial z}(E_z + \alpha)\frac{\partial T}{\partial z}]$$

$$- [v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z}] + \frac{S}{\rho c_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

Equation (14) represents the equation for temperature distribution within a body of water. It entails the basic principles of heat transfer used by other investigations, however, it is virtually impossible to solve Equation (14) without making several simplifying assumptions.

Thermal Stratification of Lakes

The preceding discussions on the heat budget and heat transfer mechanisms are germane to all studies of water temperature and models developed for predicting water temperature. The annual variation of lake temperature with discussion of the important heat transfer mechanisms will now be presented.

Initial discussion on thermal stratification will be limited to lakes since it is the most elementary body of water found in nature. Figure 3 shows an annual cycle of temperature profiles for a lake. Most lakes in the temperate zone approach isothermal conditions

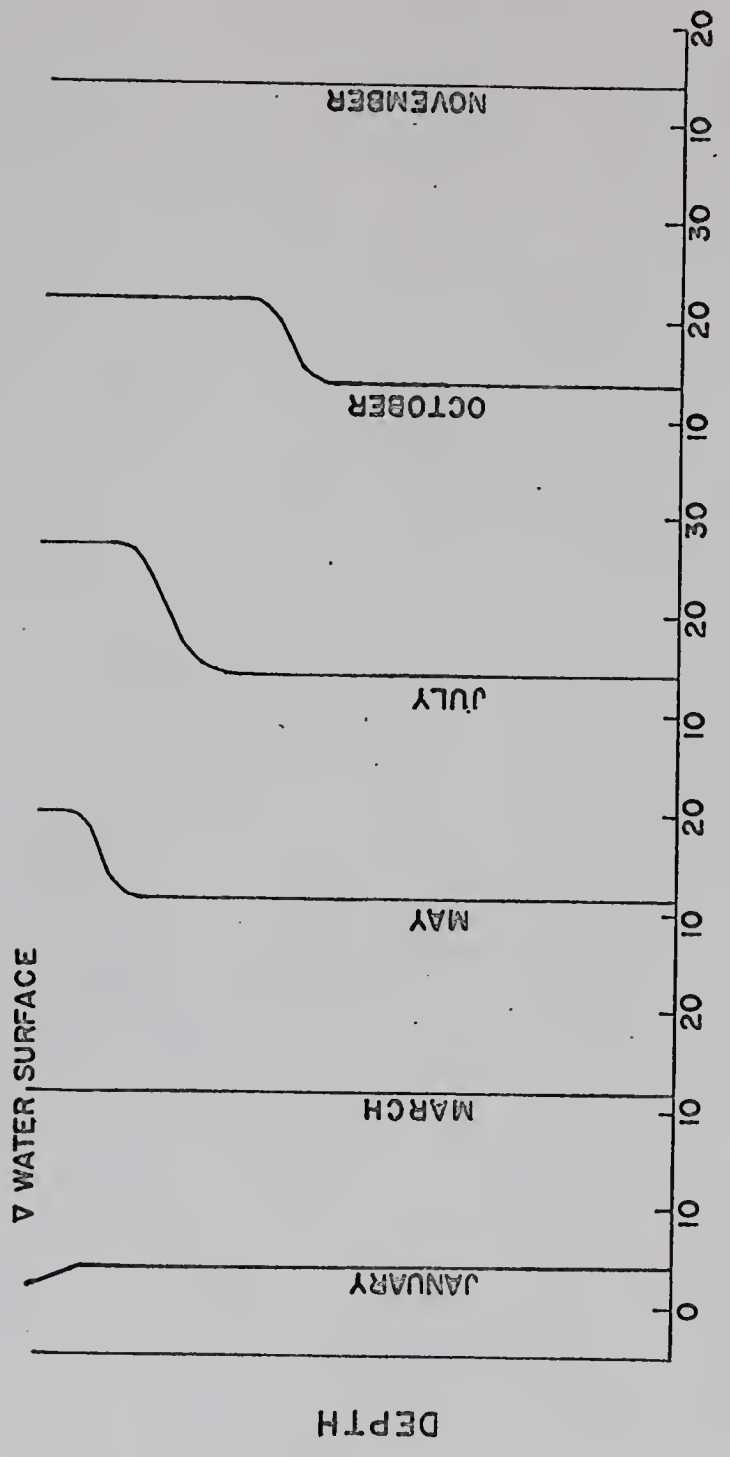


FIGURE 3. ANNUAL TEMPERATURE CYCLE for LAKES in TEMPERATE ZONE

sometime around March 21. At this time, the slightest wind will provide enough energy to mix the lake so that it is of constant quality throughout. The eddy diffusivity coefficient is constant from top to bottom. As spring begins, short-wave solar radiation is absorbed in the surface layers, warming the upper strata. At first, wind on the lake will mix warmer surface water with the lower water; but the rate of solar absorption soon exceeds the capacity of the wind for mixing, and the surface water becomes warmer than the lower water. During this early warming period, heat is transferred vertically by the eddy diffusion resulting from wind currents. Since warm water is less dense than colder water (above 4°C), the warm water will remain at the surface and form a layer of constant temperature called the epilimnion. The formation of a temperature gradient and, hence, a density gradient creates a stable condition within the lake. It would take considerable energy to mix the lake. Beneath the epilimnion, the temperature profile decreases until it reaches another layer of constant temperature. This layer is called the hypolimnion and it contains the coldest water in the lake. Since there is no temperature gradient in the hypolimnion, the water in it is easily mixed and, therefore, the quality of the water is essentially constant. The layer between the epilimnion and hypolimnion is called the metalimnion and contains the thermocline (defined as the maximum rate of decrease in temperature, $\frac{\partial^2 T}{\partial z^2} = 0$). Once the lake stratifies, the effect of eddy diffusion is limited because of the stable condition of the reservoir. Orlob[20] discusses the change in the eddy diffusivity with depth. Since the rate of heat exchange is greatest at the surface, the eddy diffusivity coefficient would be expected to be greatest

at this point. The coefficient would decrease somewhat in the epilimnion, but would still be relatively large since the epilimnion is completely mixed. The lake is most stable at the thermocline, and the eddy coefficient would be expected to be lowest at this point. It would then increase slightly to a second maximum in the hypolimnion. Figure 4 shows the change of the eddy diffusivity coefficient with depth.

As summer approaches, the epilimnion absorbs more solar radiation and the temperature of the surface layer rises, increasing the stability. The heat loss by evaporation and by back radiation prevents the temperature of the epilimnion from exceeding the mean ambient air temperature. As water evaporates at the surface, it loses its heat of vaporization and thus a small layer of cooler water is formed at the surface. During the summer evenings, when the air temperature falls below the epilimnion temperature, more heat is transferred from the water surface by evaporation, conduction, and back radiation than is absorbed in the epilimnion. In both cases, an unstable condition exists since the surface water is more dense than that below it. The slightest wind will upset this condition and set up convective currents which mix the cooler water into the epilimnion.

During the fall, the epilimnion begins to cool. The rapid decrease in the air temperature causes large transfers of heat from the epilimnion and thus sets up large convective currents. These currents go deeper and deeper into the metalimnion as the temperature gradient decreases and cause a deep layer of constant quality. The lake continually cools until sometime in November when the lake becomes isothermal and the fall "turn-over" takes place.

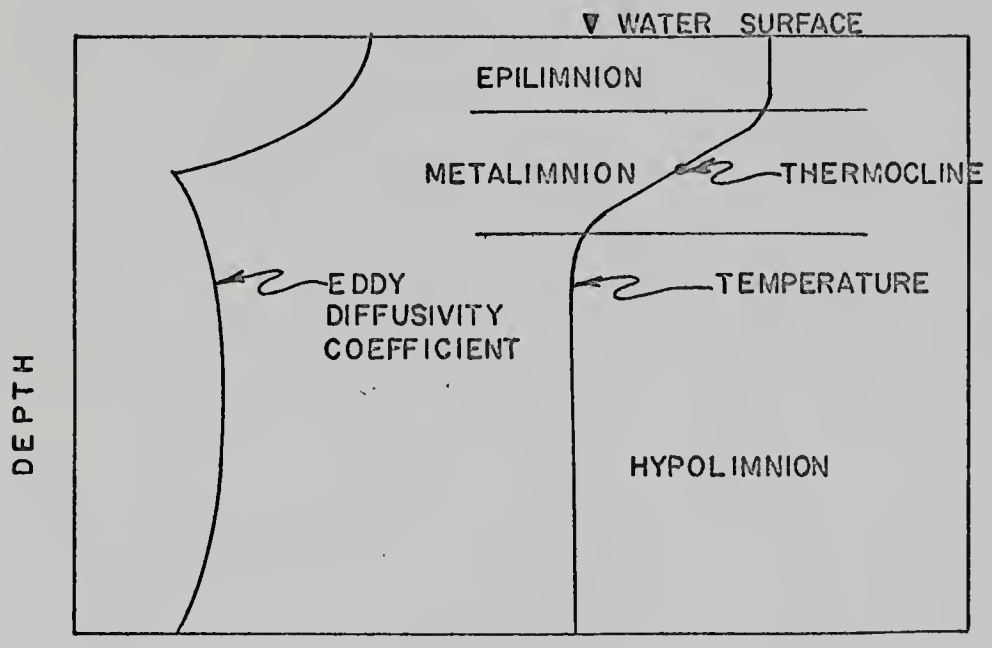


FIGURE 4. EDDY DIFFUSIVITY COEFFICIENT and TEMPERATURE vs. DEPTH

Depending upon the climate, the lake will remain essentially isothermal until March or will stratify inversely if the temperature of the water goes below 4°C. Since water is most dense at about 4°C, further cooling of the surface will cause the warmest water to be at the bottom. When winter ends and solar radiation warms the colder, upper layer to 4°C, the lake will again be isothermal and the spring "turn-over" occurs. The annual cycle is completed at that time. (Hutchinson presents a classification of lakes according to their annual temperature properties and discusses the formation of additional layers within lakes.[21])

During the annual temperature cycle of lakes, the largest portion of heat transferred is confined to the upper layer. Almost all the short-wave radiation is absorbed in the epilimnion and transfer of heat through long-wave radiation, evaporation and conduction takes place at the surface. The major modes of heat transfer are eddy diffusion from wind currents and convective currents. Dake and Harleman[22] derived an equation similar to Equation (15) for the prediction of temperature profiles in lakes. Their equation is (using the notation of this study):

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2} + \frac{S}{\rho c_p} \qquad (15)$$

Since there are no velocity components in lakes, the advection term is zero. Because of the difficulty of selecting the diffusivity coefficient, E, a priori, Dake and Harleman assumed it to be zero in their formulation. The source term, S, was a function of incident solar radiation, $(H_a - H_{ar})$ in (Btu per square foot per day) the

absorption coefficient of the fluid, η in centimeters^{-1}, and the fraction of radiation absorbed at the water surface, β_1. The source term in equation form is:

$$S = \frac{\eta(1 - \beta 1)}{\rho c_p} \quad (H_a - H_{ar}) \quad e^{-nx} \quad \ldots\ldots\ldots\ldots \quad (16)$$

The linearity of Equation (16) permitted the superposition of solutions for temperature distributions caused by surface solar absorption, internal radiation absorption, and surface evaporative loss. During the heating period, the evaporative loss at the surface caused a negative thermal gradient in the upper layer. When this occurred, Dake and Harleman mixed the epilimnion to a uniform temperature. This effect is shown in Figure 5. Even though eddy diffusivity was assumed zero, the negative temperature gradient caused by evaporation and the resultant mixing procedure could be interpreted as eddy diffusion resulting from convective cooling.

The preceding discussion of the annual temperature cycle was limited to lakes. The major difference between lakes and reservoirs is that advective heat transfer is not negligible in reservoirs whose inflows and withdrawals can have considerable effect on the temperature distribution. These effects are dependent upon several factors -- the quantity and time distribution of the inflow, the withdrawal rate, and the level of the outlet structure in the reservoir.

Density Currents

A density current is a gravity flow of a fluid of a specific density through fluid of another density. When a reservoir is

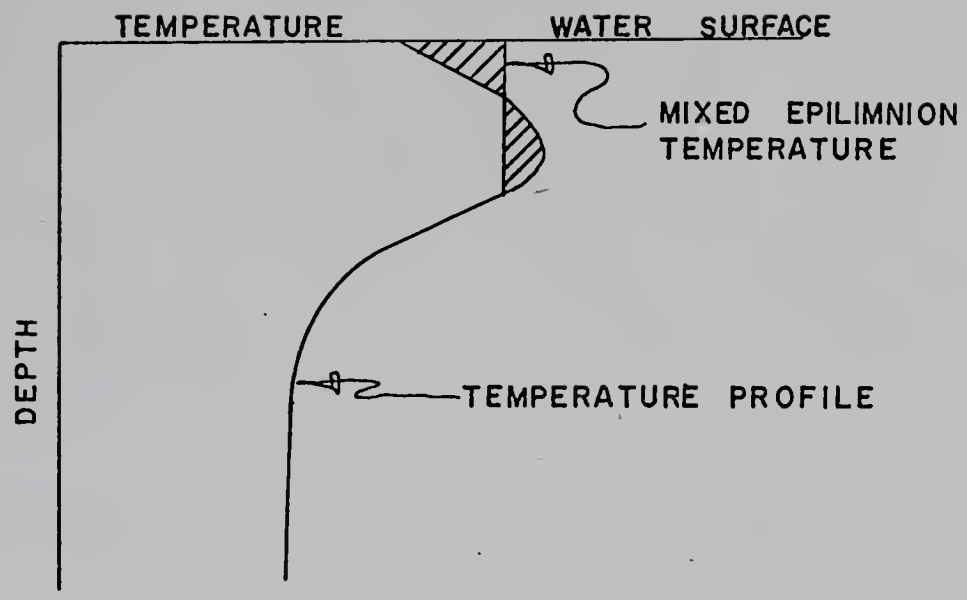


FIGURE 5. TEMPERATURE PROFILES as PREDICTED by DAKE and HARLEMAN

thermally stratified, layers of different density will be present, and, therefore, any flow of fluid will constitute a density current. Churchhill[23] reported density currents in TVA reservoirs and their effects on reservoir and stream quality. Cold water releases from deep reservoirs were observed to underflow downstream reservoirs and to be discharged from low-level outlets with only a small detention time in the reservoir. This was made possible because the incoming water was colder than that in the reservoir and was, therefore, more dense. Gravity pulled the heavier water into the lowest stratum in the reservoir creating the underflow. Since outlets from the dam were also at the level of this low stratum, the underflow moved directly through the reservoir.

Koberg and Ford[24] cited density currents at different depths within the reservoir. These currents appeared to be caused by inflow to the reservoir sinking below the surface and following the slope of the reservoir until the layer of corresponding density was reached. At this point the water spreads out horizontally causing a velocity profile within that layer in the reservoir. This phenomenon was also observed in a dye study on the Fontana Reservoir.[18] The dye was mixed into the inflow and monitored. The inflow was observed to seek a layer of approximately the same temperature. The layer containing the dye was found to be three degrees Fahrenheit higher than it was upon entering the reservoir. This was caused by mixing with warm surface water when the water entered the reservoir.

In view of these observations, most investigators assume that water withdrawn from a reservoir under stratified conditions is

selectively taken from the layer at the elevation of the outlet. The larger the density gradient near the outlet, the more confined the withdrawal layer becomes. Inflow is assumed to be confined to the layer in the reservoir of corresponding temperature.

With the background of the annual temperature cycle of lakes and the concept of density currents, the change in the downstream temperature variation, as shown in Figure 1, can now be explained. If the outlet structure is located in the hypolimnion, the colder water in the reservoir will be released first. Incoming water will seek that level in the reservoir corresponding to its temperature and will remain in that layer until the water beneath is released. Thus, incoming water is not released until it settles to the level of the outlet. The overall effect is a prolonged release of cold water from the reservoir. This has the combined effect of smoothing the annual temperature variation of released water, decreasing the average temperature of released water, and causing a shift in time of the point of maximum temperature.[11] The extent of these effects is dependent upon the level of the outlet structure, the volume of water above this level, and the rate of inflow and outflow.

If the outlet level is deep in the reservoir, more hypolimnion water must be released before the temperature will rise. The greater the volume of water above the outlet, the longer it will take to release the hypolimnion water. If the operating rule of the reservoir requires a high discharge rate in the spring and early summer, hypolimnion water will be discharged earlier and the phase shift of the regulated sine curve in Figure 1 will be less. By constructing multi-level outlets,

the reservoir operator could release water from any level in the reservoir during the stratified period and thus select that temperature which best meets downstream requirements. The methodology developed in this dissertation is based on the assumption that water can be released from any desired level in the reservoir.

Reservoir Prediction Models

Early models used to predict reservoir temperatures were quite simple. Raphael[17] proposed a model for temperature prediction in reservoirs that were well mixed during the summer months. He assumed the reservoir to be completely mixed and added the heat budget to the advected heat, in the form of inflow and outflow, to give the total heat input. This total was then distributed throughout the reservoir to produce an average reservoir temperature. Since most reservoirs undergo thermal stratification during the heating period, Raphael's model had only limited application.

Delay and Seaders[25] developed a procedure for predicting temperature profiles in proposed reservoirs. Their method was based on the use of temperature profiles from existing reservoirs of comparable size and nearby locale. Again, a procedure of this nature would have limited application.

Orlob and Selna[26] developed a mathematical model which was used to simulate reservoir thermal stratification. In their formulation they combined molecular diffusivity and eddy diffusivity coefficients into the "effective" diffusivity coefficient. "Effective" diffusivity can only be determined from thermal profile measurements made at the

reservoir. Since diffusion is a function of time and space, the measurements must be made periodically during the heating cycle. Therefore, considerable data must be obtained to determine the variation of the "effective" diffusivity coefficient for a single year. Since reservoir conditions may change from year to year because of different sequences of reservoir inputs, the "effective" diffusivity coefficient should be determined for several years in order to have a satisfactory knowledge of its behavior for a particular reservoir.[27] Orlob and Selna's model required considerable data input for application to a single reservoir. Their model could not be applied to proposed reservoirs without considerable knowledge of the "effective" diffusivity coefficient in several geographical regions.

Huber and Harleman[28] have developed a mathematical model for predicting thermal stratification in reservoirs. Their model requires knowledge of the reservoir geometry, inflow volumes and temperatures, meteorological conditions at the reservoir and outflow volumes. The model is, to some degree, an extension of Dake and Harleman's work on lakes as the eddy diffusivity is assumed to be zero. Included in the model are consideration for mixing of the inflow with reservoir surface waters before diving into the reservoir as a density current and a technique for handling the inflow and withdrawal velocity profiles as Gaussian curves. Figure 6 illustrates these velocity profiles.

Koh[29] and Kao[30] have demonstrated in the laboratory that withdrawals from a fluid under a density gradient do not come only from the layer at the elevation of the outlet, but are a composite

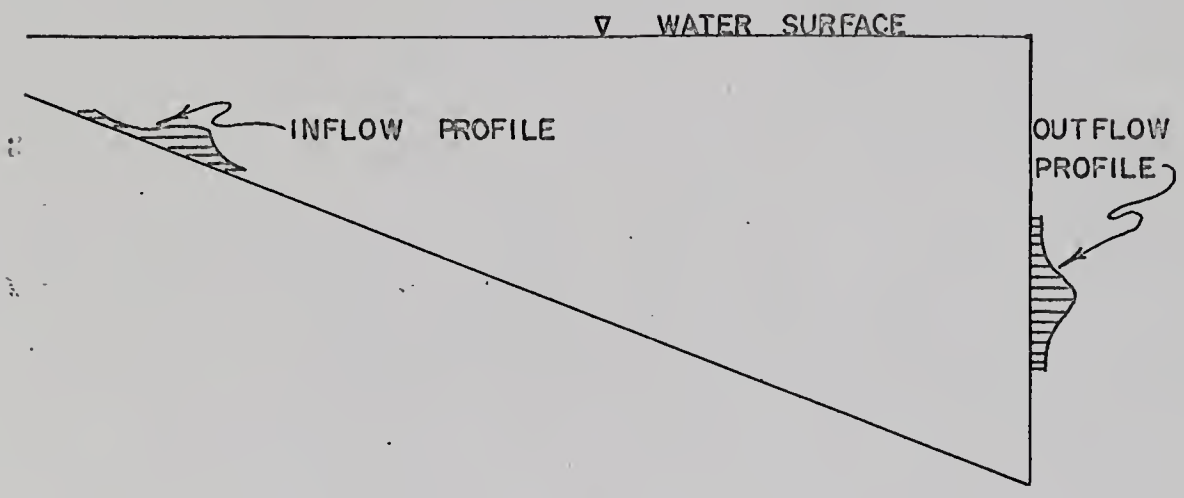


FIGURE 6. INFLOW and OUTFLOW VELOCITY PROFILE
after HUBER and HARLEMAN

of the layers in the vicinity of the outlet. The velocity profile in Koh's work was very similar to that shown in Figure 6. The degree to which the velocity profile extends into other layers is dependent upon the density gradient at the outlet. If the gradient is large, the discharge will be composed of a rather narrow swath of water from the reservoir. However, when the gradient is small, the width of the layer affected by the discharge increases.

Huber and Harleman combine the heat transfer equation with the continuity equation and the equation for continuity of mass to derive:

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial z} = \frac{\alpha}{A} \frac{\partial}{\partial z} \left(A \frac{\partial T}{\partial z}\right) + \frac{q_i (T_{in} - T)}{A} + \frac{\eta(1 - \beta_o)\phi_o e^{-\eta x}}{\rho c} \quad \ldots\ldots (17)$$

where A is the cross-sectional area of the reservoir and is a function of depth; q_i is the inflow rate per unit of vertical distance; T_{in} is the temperature of the inflow; and all other terms are as defined earlier. Equation (17) was solved by finite difference techniques on a digital computer. The equation yielded excellent laboratory agreement between observed and predicted temperatures. The model was tested on data collected by the TVA at the Fontana Reservoir. The observed temperatures at Fontana ranged from 7 - 18°C. The model yielded very good results until October when the predicted temperatures were about 2°C less than those observed. The model developed by Huber and Harleman could be used to obtain predicted temperatures for proposed reservoirs. Huber and Harleman's model was not chosen for this study due to the amount of computer time necessary for an annual cycle.

A suitable model should provide a satisfactory representation of thermal stratification and the volume and temperature of water in the

three layers of the reservoir and should be easily adapted to computation on the digital computer.

Wunderlich and Elder[19] have developed a graphical procedure for determining reservoir outflow temperatures. Their technique is based on several simplifying assumptions. First, the heat exchange between the reservoir and the atmosphere occurs only in a ten-foot layer at the surface of the reservoir. In the light of preceding discussion on convective cooling, this assumption might be improved if the depth were allowed to increase, as the heating season progresses, from, perhaps five feet in early April to twenty or twenty-five feet in July and August. This layer would correspond to the epilimnion and would have the same temperature throughout.

Second, the temperature of the ten-foot surface layer corresponds to the equilibrium temperature of the exposed body of water. This is a convenient assumption as the equilibrium temperature can be easily calculated from the heat budget, as will be explained in a later section of this chapter.

Third, each water parcel entering the reservoir spreads into a horizontal layer corresponding to its temperature. Inflows of approximately equilibrium temperature enter just below the surface layer. Field investigations show this to be approximately correct and to simplify calculations, most investigators make this assumption. The same assumption was made concerning outflows, i.e., the water withdrawn from the reservoir comes from a layer between the top and bottom of the intake.

Fourth, heat transfer by molecular and eddy diffusion is neglected below the surface layer. Since molecular diffusion is very slow and eddy diffusion is limited by the thermocline, this assumption appears to be valid.

Wunderlich and Elder applied the graphical procedure to the Fontana Reservoir in Tennessee. The observed temperatures agreed very well with the predicted temperatures except in the month of September when the predicted temperatures were about 5°F less than the observed temperature.

Ross and MacDonald[31] have applied a very similar model to reservoir projects in Montana. The major differences were the method of predicting surface water temperature and the assumed thickness of the surface layer. Ross and MacDonald used an equation similar to Equation (12) for the net rate of heat transfer at the surface. Assuming that the epilimnion was completely mixed, they expressed the time rate of change of epilimnion temperature as:

$$\frac{dTw}{dt} = \Delta HA + \frac{(c_p m_i/t)(T_i - T_w)}{c_p(m_w + m_i)} \quad \text{.....} \quad (18)$$

where T_w is the temperature of the epilimnion; ΔH is the net rate of heat transfer at the surface; A is the reservoir surface area; c_p is the specific heat of water; m_i is the change in mass of the epilimnion during the time interval dt; m_w is the mass of the epilimnion; and T_i is the temperature of the epilimnion at the beginning of the time interval. Data averaged over a seven-day period were used to calculate ΔH. The epilimnion temperature computed for the end of one time period was used as the water temperature for calculations in the following

period. This technique for calculating the surface temperature yielded results which agreed very well with observed temperatures.

Ross and MacDonald selected a surface layer depth of thirty feet for Hungry Horse Reservoir. This value was selected because of previous thermal observations at the site. They also assumed that inflowing water sought that level in the reservoir corresponding to its temperature and that withdrawals were made from the layer at the outlet. Their procedure yielded good agreement between observed and predicted temperature profiles.

These last two temperature predictive techniques lend themselves to digital computation with a minimum of machine time. The model used for predicting reservoir temperatures in this study will now be presented employing features from each of the two techniques.

Reservoir Temperature Model Used in This Study

The purpose of the reservoir model is to provide an inventory of the volume and temperature of the reservoir resulting from various operating schemes. To fulfill this purpose, it is necessary to devise a method for keeping account of the inflow and outflow from the various layers and the temperature within these layers.

Since reservoirs stratify into three layers, an inventory method was selected for adding inflows and subtracting outflows from three independent layers. Each layer was considered to have a constant temperature and to be completely mixed. Withdrawals from a particular layer during any time interval were assumed to have the quality parameter predicted for the layer at that time. Since the epilimnion

and hypolimnion have approximately constant temperature profiles, and thus a small density gradient, releases made from these layers will create constant velocity profiles as shown in Figure 7.

It seems logical to consider the hypolimnion and epilimnion as being completely mixed. However, the metalimnion has a varying temperature profile and thus a steep density gradient. Releases made under a density gradient have a velocity profile as described by Koh.[29] Withdrawals from the metalimnion will be a composite of the quality parameters in that layer and will be approximately the average of those parameters. Therefore, the release parameters from the metalimnion can be considered to be from a completely mixed layer, though actually the layer has a steep temperature gradient.

Inflows to the reservoir were considered to enter the epilimnion, metalimnion, or the hypolimnion depending upon their temperature. Inflows were added to the respective layers at the end of the time interval.

Since hypolimnion temperatures change very little during the heating season, they were assumed to remain constant throughout the period of analysis. Any underflow currents in the hypolimnion were mixed with the entire hypolimnion water. Inflows to the metalimnion were mixed with that volume present at the end of a time interval, and a new temperature determined.

Over the heating period, provision was made for increasing the epilimnion depth, which could be taken as constant or changing depending upon experience with the reservoir being considered. As the epilimnion volume increased, the "makeup" water was taken from the metalimnion and mixed into the epilimnion. The temperature of the

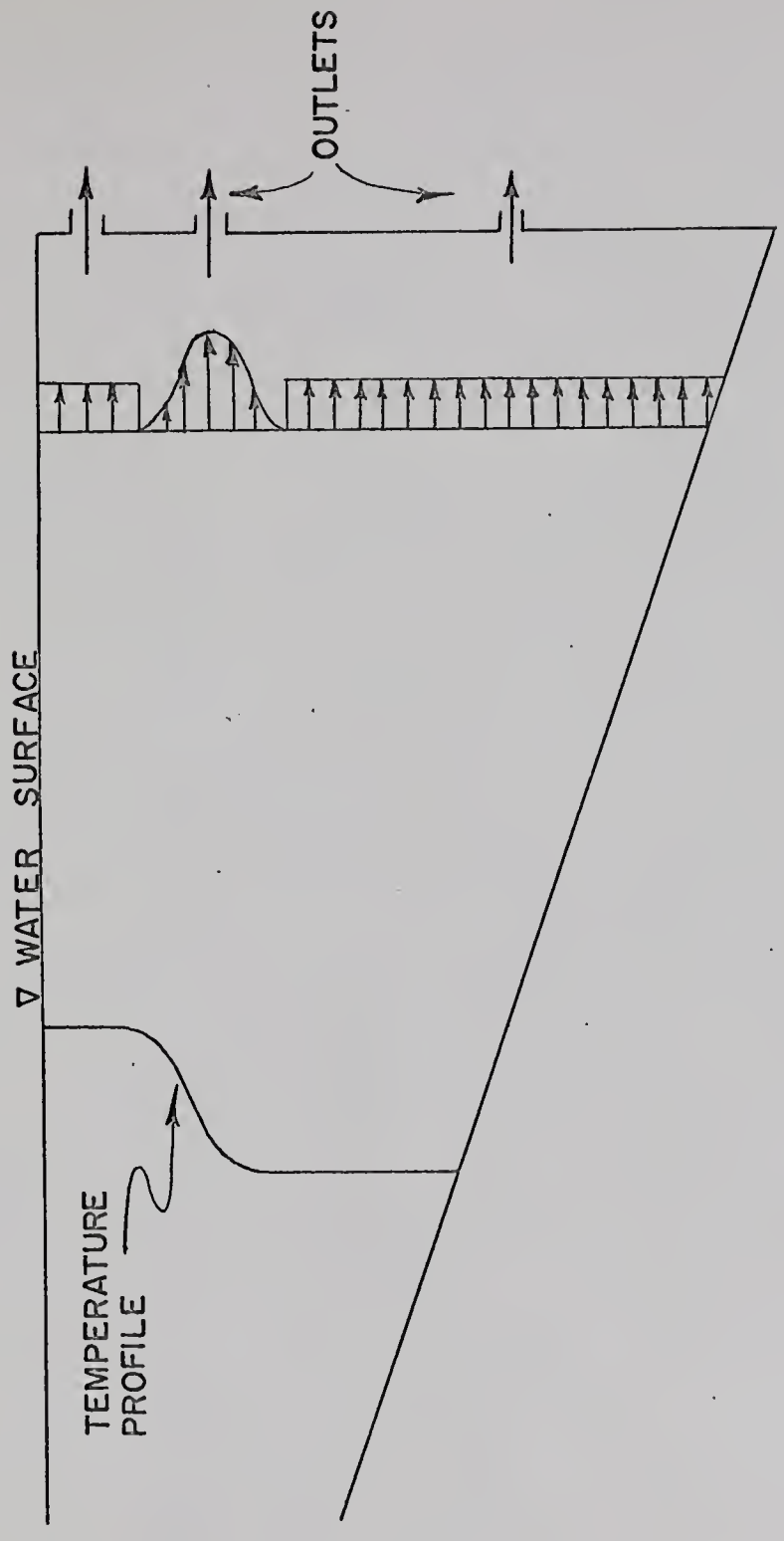


FIGURE 7. ASSUMED VELOCITY PROFILES

epilimnion was assumed to equal the equilibrium temperature. When the net rate of heat transfer at the water surface is zero ($\Delta H = 0$), the surface water temperature is considered to be at its equilibrium temperature. When the equilibrium temperature is greater than the surface water temperature, the net rate of heat transfer is positive and the surface water temperature rises. When the equilibrium temperature is less than the surface temperature, the net rate of heat transfer is negative and the water temperature decreases. Since reservoir surface temperature increases in the spring, one would expect that the equilibrium temperature is greater than the water temperature. During the summer, the surface water temperature is approximately constant, indicating that the equilibrium temperature is approximately equal to the surface temperature. In the fall, the surface water temperature decreases; therefore, the equilibrium temperature must also be decreasing. Since the surface temperature is always striving toward the equilibrium temperature, the equilibrium temperature should be a relatively good predictor of the surface water temperature. The following development of the equilibrium temperature is according to Edinger and Geyer.[14]

If in Equation (12) the surface water temperature, T_s, is equal to the equilibrium temperature, T_E, then the net rate of heat transfer, ΔH, is zero and the equation becomes:

$$H_R = \sigma\gamma_w(T_E+460)^4 + (a+bW)(e_E-e_a) + .26(a+bW)(T_E-T_a) \quad \ldots\ldots\ldots \quad (19)$$

where T_E and e_E have been substituted for T_s and e_s, respectively. Subtracting Equation (20) from Equation (12) eliminates H_R:

$$\Delta H = -\{\gamma_w \sigma[(T_s+460)^4 - (T_E+460)^4] + (a+bW)(e_s-e_E) + (a+bW)\ 0.26\ (T_s-T_E)\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

Edinger and Geyer use a linear relationship to approximate saturation vapor pressure and water temperature for ten-degree Fahrenheit temperature ranges. The constants for the temperature increments are shown in Table 1.

TABLE 1

PROPORTIONALITY COEFFICIENTS AND INTERCEPTS FOR THE TEMPERATURE, VAPOR PRESSURE APPROXIMATION [14]

Temperature Range, °F	β, mm-Hg/°F	Intercept C (β)mm-Hg
40-50	.2910	-5.47
50-60	.4050	-11.22
60-70	.5553	-20.15
70-80	.6667	-27.80
80-90	.9900	-53.33
90-100	1.289	-89.28

The term β in Table 1 is the proportionality coefficient in the equation below:

$$(e_s - e_E) = \beta\ (T_s - T_E) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

The expressions to the fourth power in Equation (20) were approximated by the binomial expansion, keeping the first and second order terms. The approximation is:

$$\sigma\gamma_w(T_s+460)^4 = \sigma\gamma_w 460^4 \left[1 + \frac{4T_s}{460} + 6\left(\frac{T_s}{460}\right)^2\right] \quad \ldots\ldots\ldots\ldots\ldots \quad (22)$$

Substituting for σ and γ_w and including the approximations Equation (20) becomes:

$$\Delta H = - [15.71 (T_s - T_E) + .051(T_s^2 - T_E^2) + (a+bW)\ \beta(T_s - T_E) + 0.26\ (a+bW)(T_s - T_E)] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

Combining like terms yields:

$$\Delta H = - [15.7 + (0.26+\beta)(a+bW)] (T_s - T_E) + .051(T_s^2 - T_E^2) \quad \ldots\ldots \quad (24)$$

From Equation (24), Edinger and Geyer define the exchange coefficient, K in Btu per square feet per day per degree Fahrenheit, as:

$$K = 15.7 + (.026 + \beta)(a+bW) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (25)$$

Using the binomial approximation for the fourth power term in Equation (19), and the expressions:

$$a+bW = \frac{K - 15.7}{0.26 + \beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (26)$$

and,

$$e_E = \beta\ T_E + C\ (\beta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (27)$$

Edinger and Geyer reduced Equation (19) to:

$$T_E + \frac{.051 T_E^2}{K} = \frac{H_R - 1801}{K} + \frac{K - 15.7}{K} \left[\frac{e_a - C(\beta)}{0.26+\beta} + \frac{0.26 T_a}{0.26+\beta}\right] \quad \ldots\ldots\ldots \quad (28)$$

By estimating an initial temperature range for the constants β and C (β), Equation (28) can be solved for the equilibrium temperature. T_a, e_a, H_R and K are determined from meteorological data. If the equilibrium temperature falls within the assumed range used for β and C (β), the calculation is complete. Otherwise, the calculation must be repeated until T_E falls within the estimated range.

Computer programs have been coded for calculating the epilimnion temperature using Equation (28) and for keeping the inventory of water volume and temperature in the three reservoir layers. The program listings are in the Appendix with appropriate flow charts and Glossary of terms.

The next section will discuss the effects of reservoirs on dissolved oxygen and the technique employed in this study to account for these effects.

Dissolved Oxygen

Dissolved oxygen levels in natural streams, with little or no pollution, are generally near saturation, which depends upon the temperature and chloride content of the stream. However, with the introduction of a reservoir on a stream, dissolved oxygen concentration of releases may range from 0 to 100% of saturation.[32] This alteration in dissolved oxygen levels is the result of many physical, chemical, and biological processes taking place within the reservoir. The more important processes will be discussed in this section along with recent attempts at improving reservoir water quality.

Sources of Dissolved Oxygen

One of the principal sources of oxygen in natural waters is atmospheric oxygen that is absorbed into the water across the air-water interface. Oxygen absorbed at the water surface is diffused downward by wind-induced turbulence. The rate of oxygen absorption is proportional to the difference between the saturation level and the dissolved oxygen present:

$$\frac{dC}{dt} = K_2 (C_s - C) \quad \text{.....} \quad (29)$$

where C is the dissolved oxygen in milligrams per liter (mg/l); K_2 is the reaeration coefficient in $days^{-1}$; and C_s is the saturation level of dissolved oxygen in milligrams per liter. The saturation values for pure water can be expressed as:[33]

$$C_s = 14.652 - 0.41022T + 0.0079910T^2 - 0.000077774T^3 \ldots\ldots \quad (30)$$

where T is temperature in degrees Centigrade.

The reaeration coefficient is dependent upon the turbulence and temperature of the water. Expressions for the reaeration coefficient and its relation with temperature will be given in Chapter III. For lakes, the reaeration coefficient has been reported as ranging from 0.05 to 0.10 day^{-1} (see Babbitt and Baumann[34]).

Since dissolved oxygen is diffused into a body of water by wind action, thermal stratification in reservoirs prevents atmospheric oxygen from being transferred to the metalimnion and hypolimnion. The absence of stratification in the epilimnion virtually insures saturation-dissolved oxygen levels within the epilimnion.

Photosynthesis by phytoplankton and aquatic plants is another source of oxygen in water. Since sunlight is the catalyst for photosynthesis, oxygen produced by this process is usually confined to the layer absorbing the most sunlight. In reservoirs, this layer is the epilimnion. Depending upon the concentration of phytoplankton, these organisms can be an oxygen credit or debit. In the production of oxygen, phytoplankton also respire, thus consuming oxygen. Large concentrations of phytoplankton, "blooms," have been observed to almost deplete oxygen from the water during night hours. Hutchinson[21] points out a case in which the oxygen varied from 18.7 mg/l in the afternoon to 2.2 mg/l in the early morning.

An important source of oxygen to the lower layer of a stratified reservoir is inflowing water. If the oxygen content of the inflow is larger than that at the level it enters the reservoir, the total oxygen level will increase in that layer.

Dissolved Oxygen Sinks

The primary sinks of dissolved oxygen are chemical oxidation of inorganic compounds and biological respiration. The presence of oxidizable inorganic compounds may be from surface inflows, underground inflows, sediments and industrial pollution. In the presence of oxygen, compounds such as sulfide and nitrite are oxidized to sulfate and nitrate. Reservoirs which have large concentrations of these compounds may have very little oxygen in the lower strata.

Biological respiration in reservoir waters may be from animals, phytoplankton, zooplankton, or bacteria. Since fish require oxygen to survive, they will certainly exert a demand on the oxygen resources of a reservoir; however, the relative amount of oxygen consumed by fish is probably very small.

Dissolved organic compounds are biochemically oxidized by bacteria in the reservoir. The bacteria consume oxygen as they degrade the organic material. The rate at which oxygen is consumed is directly proportional to the amount of organic material present. In some lakes, this rate has been observed to range from 0.07 to 0.28 mg/l/day.

The zooplankton and phytoplankton are sometimes interdependent. Zooplankton consume oxygen as they feed on phytoplankton and bacteria in the water. Naturally, large algal growths will spawn large populations of zooplankton and hence create an even larger oxygen sink during night hours.

Dissolved Oxygen Profiles

The sources and sinks of oxygen combine to cause a variation in dissolved oxygen with depth and time. This effect is shown in Figure 8

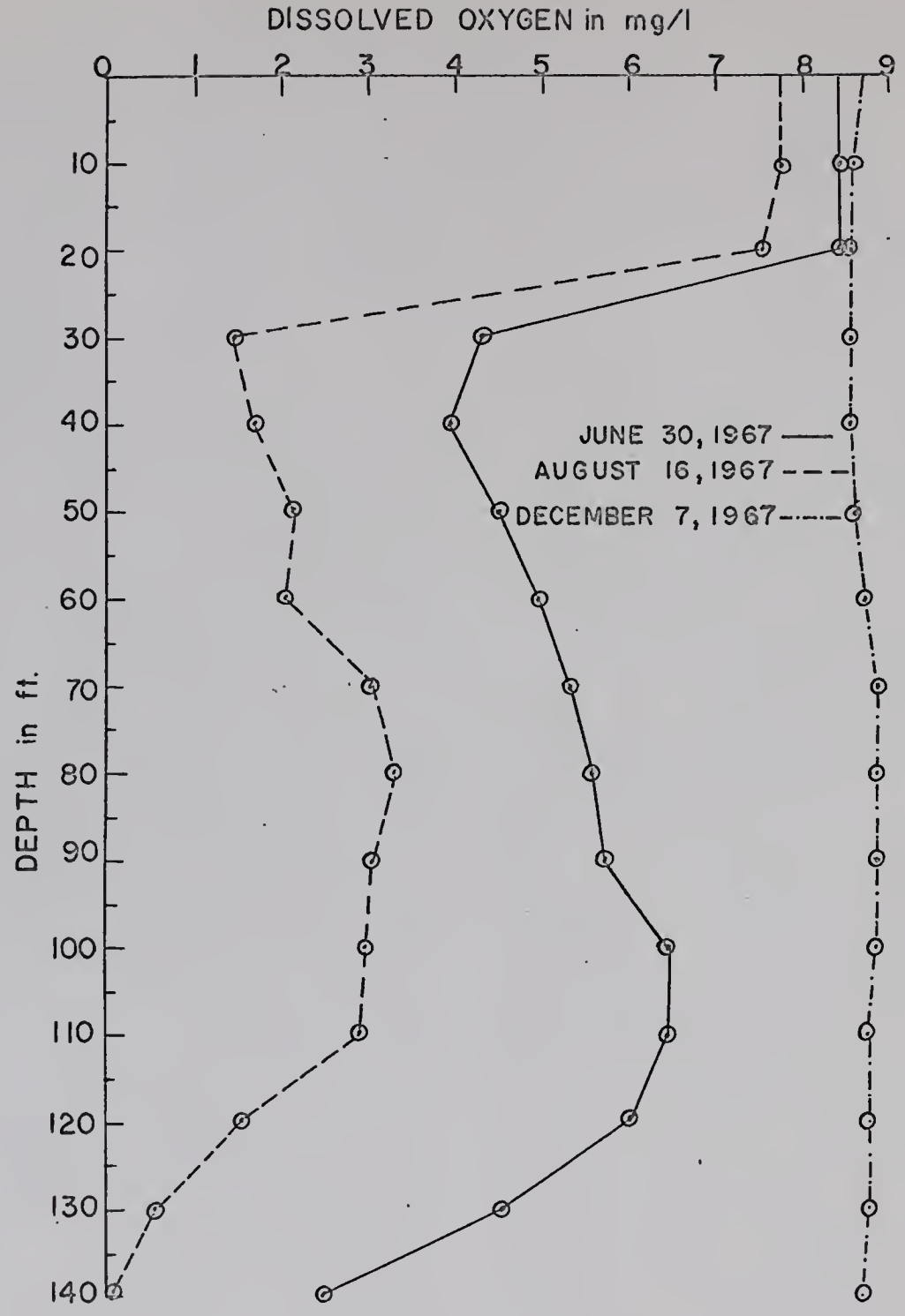


FIGURE 8a. DISSOLVED OXYGEN VARIATION with TIME and DEPTH

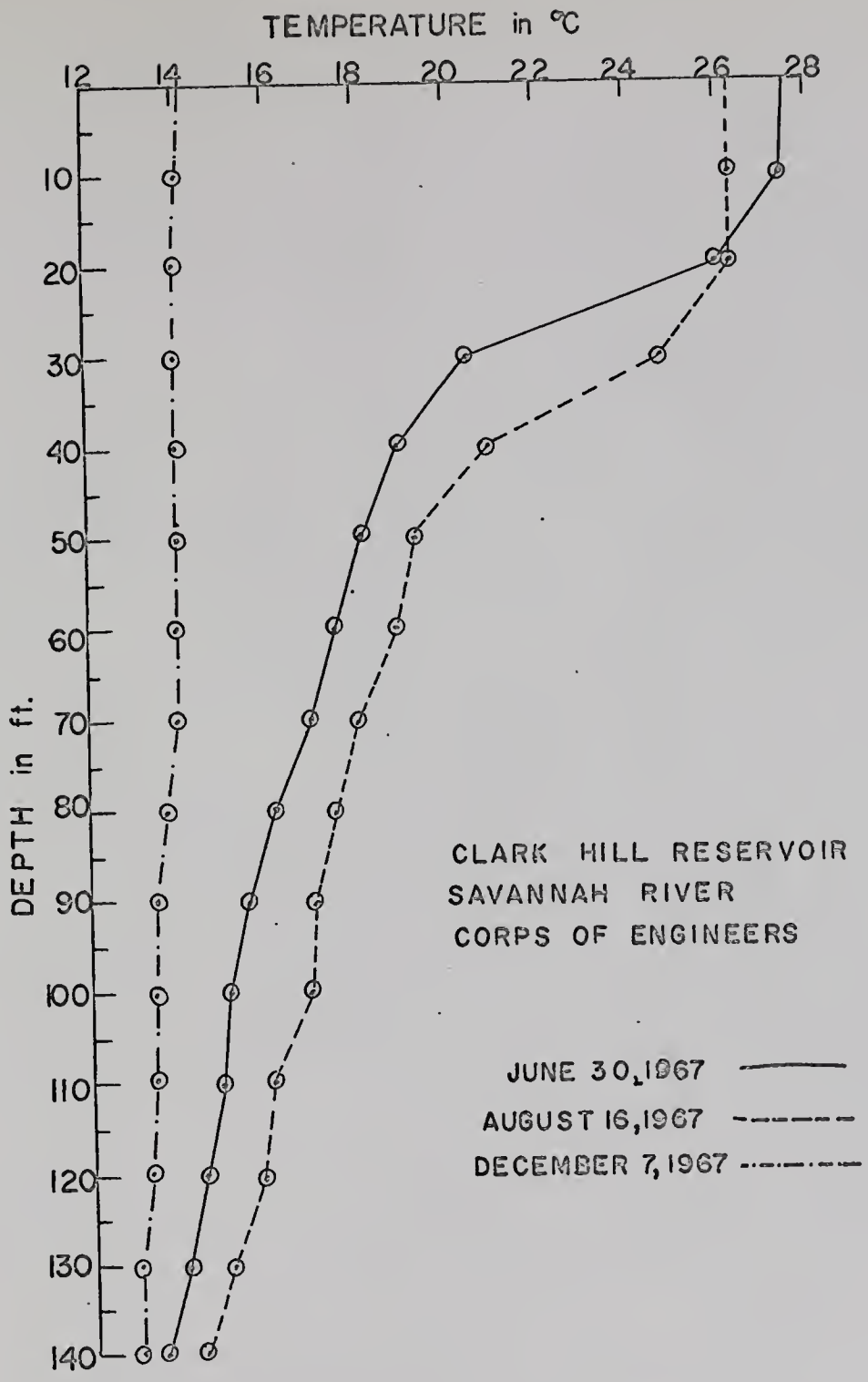


FIGURE 8b. TEMPERATURE VARIATION with TIME and DEPTH

with corresponding temperature profiles for data collected on the Clark Hill Reservoir in Georgia. The dissolved oxygen is seen to stratify as the reservoir undergoes thermal stratification. The oxygen concentration is constant in the epilimnion, decreases in the metalimnion, and increases in the hypolimnion before reaching a minimum at the bottom. The high concentration in the epilimnion is the result of surface aeration and phytoplankton. As mentioned earlier, the oxygen sources are confined to the epilimnion with little, if any, oxygen being diffused to the lower layer.

The point of lowest dissolved oxygen occurs in the metalimnion, which is not uncommon and is termed the metalimnetic minimum.[21] The most widely known theory of the minimum is that it represents a region into which dead phytoplankton, organic material, and zooplankton have settled from the epilimnion. It is believed that the increasing viscosity of the water in the metalimnion prevents this suspended material from settling into the hypolimnion. The result is a large growth of bacteria and zooplankton consuming the oxygen as they feed on the dead phytoplankton. Churchill and Nicholas[35] report data collected on the Boone Reservoir in Tennessee which seem to corroborate this theory. Profiles of zooplankton counts in the reservoir show a maximum of zooplankton at the metalimnetic minimum.

The dissolved oxygen in the hypolimnion water of Figure 8a decreases from June 30, 1967, to August 16, 1967. Evidently there is oxidizable organic material being degraded by bacteria. At a depth of one hundred ten feet, the rate of oxygen depletion is 0.077 mg/l/day, which is in agreement with Hutchinson.

By December 7, 1967, the dissolved oxygen concentration is uniform in the reservoir. This is to be expected after the fall turnover. If another profile were shown for March or April, it probably would show very little thermal gradient and uniform dissolved oxygen concentration.

Improving Release Water Quality

Kittrell[36] discussed several alternative methods for improving water quality in released water. The most promising methods were multi-level penstock intakes, turbine reaeration, and reservoir mixing.

Multi-level intakes permit the selection of that quality of water which best meets downstream needs. The installation of multi-level penstock intakes adds considerable expense to the reservoir capital costs. However, there is very little operating expense and benefits from increased flexibility in reservoir management to meet downstream quality requirements may offset the capital cost. It should be pointed out that no methodology exists at this time for estimating the benefits accruing from multi-level intakes, but the methodology developed in this study will provide a basis for comparison with single outlets.

Turbine reaeration is easily employed at power installations where the turbines are higher than the tailwater level, creating a substantial vacuum at low flows. Vents opened to the atmosphere introduce large quantities of air in the discharge water. The loss of power when the vent is opened may range from 2.5 to 10.6 % of the total power being generated. The loss is inversely proportional to the head on the turbines. In a test case of several hundred feet of head, the

power loss was negligible.[37] Oxygen increases may range from 5000 to 10,000 lbs/day/1000 cfs.[38]

Since the work of Symons et al.[39] and Irwin et al.,[40] much attention has been given to reservoir mixing as a means for controlling water quality. It was shown that pumping water from the deepest portion of small lakes would effectively modify the thermal gradient and increase the dissolved oxygen at lower depths. In short, the quality of the lakes was improved. The Corps of Engineers in the South Atlantic Division has completed one year of work on the first attempt to destratify a reservoir.[32] Air diffusers were used at one location in Alatoona Reservoir in Georgia to disrupt thermal stratification and improve dissolved oxygen concentrations. The Corps is now in the process of evaluating the data collected in the summer of 1968. No one has yet determined the benefits of reservoir mixing, but a reasonable comparison can be made from the methodology developed in this study.

Reservoir mixing causes warming of hypolimnion water in the reservoir. If the reservoir is completely mixed for the entire heating season, then the cool hypolimnion water will not be available for downstream fisheries or for cooling water in steam-electric power plants. This could be a severe detriment to overall basin economy and should be included in analysis of reservoir mixing.

In the preceding discussion, the effects of reservoirs on dissolved oxygen have been outlined and some methods of improving water quality have been mentioned. The next section will outline the method employed in this study to predict dissolved oxygen in reservoir releases.

Method Used in This Study

In several water quality studies (e.g., Bramhall and Mills,[7] Davis,[5] Worley et al.,[41] Feigner[42]) little, if any, consideration is given to temperature and dissolved oxygen variation in reservoir releases. Bramhall and Mills, as well as Davis, make no mention at all of reservoir water quality in their evaluation of low-flow augmentation. Worley et al. assume the dissolved oxygen of reservoir releases to be at the saturation level. Feigner does evaluate the low-flow augmentation requirements due to arbitrary changes in temperature, but does not attempt to control temperature in the releases at the reservoir.

Probably the reason few attempts have been made to account for the dissolved oxygen variation is that little work has been done to develop a predictive model for dissolved oxygen in reservoirs. Due to the complex biological processes taking place in reservoirs and the variation in the processes from time to time and from reservoir to reservoir, it is very difficult to formulate a dissolved oxygen model which can be applied to reservoirs. In view of the previous discussion on the effects of dissolved oxygen, assuming dissolved oxygen saturation in the release water would be unrealistic and would result in a bias in allowable waste loads. Saturation levels in releases probably would be maintained in the stream until waste loads are discharged. With low dissolved oxygen releases, the stream will receive oxygen through reaeration; but, depending upon the degree of turbulence in the stream and time of travel to the point of waste discharge, the dissolved oxygen level may be much less than saturation.

Therefore, the waste loadings in the saturated release may be much larger because of total oxygen resources at the point of waste discharge.

In an attempt to better represent the dissolved oxygen content of released water, a method for permitting dissolved oxygen to vary in time and among layers is employed. This method consists of using a matrix of coefficients which vary both vertically and horizontally. The variation of the coefficients in the vertical represents the change in the percentage of saturation for each time period. The horizontal variation represents the change among layers. Figure 9 illustrates the formation of this matrix.

Residence Time	E	M	H
t=1	a_{11}	a_{12}	a_{13}
2	a_{21}	a_{22}	a_{23}
3	a_{31}	a_{32}	a_{33}
.	.	.	.
.	.	.	.
.	.	.	.
n	a_{n1}	a_{n2}	a_{n3}

FIGURE 9. MATRIX FOR REPRESENTING DISSOLVED OXYGEN VARIATION WITHIN THE RESERVOIR

An example of the use of this matrix might be that in the first time interval the reservoir is completely mixed with dissolved oxygen at saturation. For this interval $a_{11} = 1.00$, $a_{12} = 1.00$, and $a_{13} = 1.00$. In the second time interval the dissolved oxygen might vary as follows: 95% saturation in the epilimnion, 80% saturation in the metalimnion, 85% saturation in the hypolimnion. Then the second row

of coefficients is $a_{21} = .95$, $a_{22} = .80$, and $a_{23} = .85$. The matrix is filled in this manner from the first time interval until the nth interval, which in this study represents the last time interval.

The use of this matrix permits the calculation of dissolved oxygen within the reservoir given the temperature of the water in each layer. A method of this type can be adapted very easily to data taken on existing reservoirs and can be employed in the analysis of proposed reservoirs to see what levels of dissolved oxygen in the reservoir cause downstream water quality problems. As simple as it may be, this method is an improvement over previous work in which no variation in dissolved oxygen is assumed.

In this chapter, the effects of impoundments on temperature and dissolved oxygen have been discussed. The marked variation of temperature and dissolved oxygen in time and space were illustrated by Figures 3 and 8. Discussion was presented on the causes of these variations, models used to describe them, and the methods employed in this study to predict temperature and dissolved oxygen changes. In addition, alternatives for improving water quality in reservoir releases were mentioned, and it was stated that the methodology developed in this study will provide a means for comparing these alternatives.

Upon release from the reservoir, temperature and dissolved oxygen do not remain constant, but vary as the stream flows through the basin. In Chapter III, the models used in this study to describe the variation in temperature and dissolved oxygen along the stream profile will be presented.

III. STREAM TEMPERATURE AND DISSOLVED OXYGEN MODELS

In Chapter II, the models used in this study to predict the effects of reservoirs on temperature and dissolved oxygen were presented. Since this study deals with a reservoir-river system, it is necessary to develop mathematical models to predict the downstream variation of the water released from the reservoir. The changes in stream quality will be discussed in this chapter, and mathematical models used to describe these changes will be derived.

Stream Temperature

Since the water in the deeper layer of the reservoir is usually at a low temperature, upon release it will approach the equilibrium temperature determined by meteorological conditions. The rate at which the water approaches equilibrium will depend upon the difference between the water temperature and the equilibrium temperature, the wind over the river, and the area of the stream water surface.

In the previous chapter, the concept of equilibrium temperature was discussed and it was stated that water temperature is continually approaching equilibrium. The difference between the equilibrium temperature and the water temperature is the driving force for the rate of change of water temperature. When the difference between the equilibrium and water temperature increases, the rate at which the water temperature changes will also increase.

The rate of temperature change also depends upon the wind. If the water temperature is increasing, meteorological conditions are such that the equilibrium temperature of the water is greater than the in situ water temperature, causing the water to absorb heat. Since the heat is provided from the air over the water, the more air brought into contact with the water, the faster the water temperature will increase. The source of air movement is wind; therefore, the greater the wind velocity, the faster the rate of change of water temperature. A large percentage of the heat exchange between the stream and the overlying atmosphere takes place at the water surface. If the water surface is small in comparison to the total volume of the stream, the stream will exchange heat with the air at a slower rate than if the surface were larger.

Heat exchange in streams is affected by thermal stratification in the stream. Thermal stratification is caused by a rapid warming of the surface and by high temperature discharges into the stream. The thermal gradient may remain in sluggish streams, but it is usually destroyed by turbulent mixing in swiftly flowing streams. To make calculations easier, complete mixing usually is assumed.

Model Used in this Study

The derivation of the stream temperature model will be after Edinger and Geyer.[14] It is convenient to begin with Equation (14):

$$\frac{\partial T}{\partial t} = \left[\frac{\partial}{\partial x}\left(E_x + \alpha\right)\frac{\partial T}{\partial x}\right] + \left[\frac{\partial}{\partial y}\left(E_y + \alpha\right)\frac{\partial T}{\partial y}\right] + \left[\frac{\partial}{\partial z}\left(E_z + \alpha\right)\frac{\partial T}{\partial z}\right]$$

$$- \left[v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z}\right] + \frac{S}{\rho c_p} \quad \ldots\ldots\ldots\ldots\ldots \quad (31)$$

where T is temperature in degrees Fahrenheit; t is time in days; E_x, E_y, and E_z are coefficients of eddy diffusivity in the x, y, and z directions in square feet per day; α is the coefficient of molecular diffusivity in square feet per day; v_x, v_y, and v_z are velocity components in the x, y, and z directions in feet per day; and S is the external heat source in Btu per cubic feet per day.

The x-direction is along the length of the stream, the z-direction is vertical, and the y-direction is laterally across the stream. In most stream temperature models, the first assumption is that the stream can be considered to be completely mixed (e.g. Raphael,[18] Duttweiler,[43] and Velz and Gannon[44]).

If a stream is completely mixed, there will be no lateral or vertical gradients of temperature; i.e., $\frac{\partial T}{\partial y} = 0$, $\frac{\partial T}{\partial z} = 0$. Therefore, two diffusion terms and two advection terms become negligible. This leaves:

$$\frac{\partial T}{\partial t} = [\frac{\partial}{\partial x} (E_x + \alpha) \frac{\partial T}{\partial x}] - v_x \frac{\partial T}{\partial x} + \frac{S}{\rho c_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (32)$$

For the purposes of this study, two additional conditions will be required in the stream. First, the temperature at any point along the stream will remain constant during the time interval for which the temperature is predicted. This is the condition of steady-state, $\frac{\partial T}{\partial t} = 0$. A condition of this nature will negate diurnal temperature variations. Secondly, the primary mode of heat transfer will be advection. Under this condition, the heat transferred by diffusion will be considered negligible when compared with advected heat. Thus, the heat source term is balanced by the heat advection term:

$$v_x \frac{\partial T}{\partial x} = \frac{S}{\rho c_p} \quad \text{.....} \quad (33)$$

Taking the intergral of Equation (33) over the depth, z, yields:

$$\int_o^d v_x \frac{\partial T}{\partial x} \, dz = \int_o^d \frac{S}{\rho c_p} \, dz \text{.....} \quad (34)$$

For the purposes of this study, all heat exchanged into or out of the body of water is assumed to go through the surface; hence, the term $\int_o^d S \, dz$ is the net rate of heat transfer, ΔH. The term $\int_o^d v_x \, dz$ can be thought of as the stream flow per unit width of stream. Then, Equation (34) becomes:

$$\rho c_p \, d_m \, v_x \frac{\partial T}{\partial x} = \Delta H \quad \text{.....} \quad (35)$$

where d_m will be taken as the mean depth of the stream. If in Equation (24) of Chapter II the second order terms are neglected, the net rate of heat exchanged becomes equal to the product of the exchange coefficient and the difference between the water temperature and the equilibrium temperature:

$$\Delta H = - K (T_s - T_E) \quad \text{.....} \quad (36)$$

where K is the exchange coefficient defined in Equation (25) in Btu per square feet per degree Fahrenheit per day:

$$K = 15.7 + (0.26 + \beta) (a+bW) \quad \text{.....} \quad (37)$$

Substituting Equation (36) into Equation (35) and solving for the rate of change in temperature in the downstream direction yields:

$$\frac{\partial T}{\partial x} = \frac{- K (T_s - T_E)}{\rho c_p \, d_m \, v_x} \quad \text{.....} \quad (38)$$

Comparing Equation (38) with previous discussion on the change in temperature reveals that the change in the longitudinal gradient is

a function of the driving force, $(T_s - T_E)$, the exchange coefficient, K, the mean depth and stream velocity. Further scrutiny shows that the wind velocity is present in the exchange coefficient,and the surface area of the stream is found in $d_m v_x$ by substituting in the continuity equation for velocity:

$$d_m v_x = d_m \frac{Q}{A} = \frac{d_m Qx}{d_m Bx} = \frac{Qx}{A_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (39)$$

where x is distance downstream in feet; A_s is stream surface area in square feet; A is average cross-sectional area in square feet; and Q is the stream flow in cubic feet per second; and B is the average width in feet. Thus it can be seen that all the factors which were said to influence the rate of change of the release water are present in Equation (38). Integrating Equation (38) between the limits x = 0 and $x = x_1$ yields the predictive equation used in this study.

$$T_f = (T_i - T_E)\, e^{-r_2} + T_E \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (40)$$

where T_f is the temperature at x_1 in degrees Fahrenheit; T_i is the temperature at x = 0 in degrees Fahrenheit; and

$$r_2 = \frac{K x_1}{\rho c_p d_m v_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (41)$$

If the river downstream from the reservoir is divided into several reaches, Equation (40) can be used successively in each reach to calculate T_f at the end of the reach,with T_i being the temperature at the end of the previous reach. The reaches should be divided in such a manner that the flow and hydraulic properties of the stream can be considered as constants within the reach. To use Equation (40) one must have available meteorological data to determine the equilibrium temperature and the exchange coefficient. The temperature predicted

by Equation (40) is the average temperature during the time interval for which the stream flow and meteorological data are considered constant. The shorter the time intervals between data collection, the closer the estimate becomes to observed stream temperature behavior. However, temperature predicted from monthly data should be approximately equal to the mean monthly temperature. It should be noted that predictions for reaches whose travel times are longer than the intervals between data collection can not be made because steady-state conditions would not exist. The data averaging interval, the period for which the data can be assumed approximately constant, can not be less than the time for which the equation is assumed valid or the steady-state condition is violated.

Dissolved Oxygen

The sources and sinks of dissolved oxygen in reservoirs were discussed in Chapter II. It was pointed out that almost no methodology exists for predicting dissolved oxygen in reservoirs. However, in the case of streams, several attempts have been made to develop predictive equations for dissolved oxygen. The equations estimate the dissolved oxygen as a function of various parameters, which are determined from data collected at the river or are assumed when data are unavailable. The parameters generally are taken as constant throughout the reach of the river to which the equation is applied. The estimated values often disagree with observed values with the result that little reliability can be placed on the estimated oxygen sag curve. One of the major reasons for this discrepancy is that biological regimes are assumed to be constant, when actually they vary among rivers and even within the

same river. Hence, there is an increasing need for a predictive model which reflects the uncertainties of the dissolved oxygen balance in streams.

Streeter and Phelps[45] derived the most frequently used model for predicting the stream dissolved oxygen profile. In their model, the only oxygen sink was the utilization of oxygen by bacteria during the decomposition of dissolved and suspended organic material. The rate at which the organic material was oxidized was assumed proportional to the concentration of the organic material. The equation used to represent this process was the first order kinetic equation:

$$\frac{dL}{dt} = -K_1 L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (42)$$

where L is the biochemical oxygen demand (BOD) in milligrams per liter at time t and K_1 is the deoxygenation coefficient in $days^{-1}$. Integration of Equation (42) yields:

$$L = La \; e^{-K_1 t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (43)$$

where La is the ultimate BOD in milligrams per liter.

The only source of oxygen in the Streeter-Phelps equation was surface reaeration. The equation for the rate of dissolved oxygen transfer was also expressed in a first order equation as:

$$\frac{dD}{dt} = -K_2 D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (44)$$

where D is the dissolved oxygen deficit in milligrams per liter and K_2 is the reaeration coefficient in $days^{-1}$. The dissolved oxygen deficit is expressed as the difference between the dissolved oxygen saturation value, C_s in milligrams per liter, and the actual dissolved oxygen, C in milligrams per liter:

$$D = C_s - C \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (45)$$

The reaeration coefficient was determined to be proportional to the stream velocity and inversely proportional to the stream depth. Their work on the reaeration coefficient has been confirmed by several authors with only minor changes in exponents and constants. A recent formula for the reaeration rate was presented by Langbein and Durum:[46]

$$K_2 = 3.3 \frac{v}{d_m^{1.33}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (46)$$

where d is the average depth in feet and v is the stream velocity in feet per second.

Streeter and Phelps combined the oxygen source and sink into the equation for the rate of change of dissolved oxygen deficit:

$$\frac{dD}{dt} = K_1 L - K_2 D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (47)$$

Integrating Equation (48) yields the classical Streeter-Phelps Equation:

$$D = \frac{K_1 La}{K_2 - K_1} (e^{-K_1 t} - e^{-K_2 t}) + Da\, e^{-K_2 t} \quad \ldots\ldots\ldots\ldots \quad (48)$$

where Da is the initial dissolved oxygen deficit in milligrams per liter. The dissolved oxygen sag curve, as predicted by Equation (48), is shown in Figure 10 . The curve shown is typical of the oxygen profile below waste discharges. The accuracy with which the Streeter-Phelps Equation predicts the sag curve depends upon many factors; among them are: the selected deoxygenation coefficient, K_1; the variation of the reoxygenation coefficient, K_2; other sources of deoxygenation such as phytoplankton respiration and benthol demand; and oxygen production from photosynthesis.

Variation of K_1 and K_2

Streeter and Phelps determined that the deoxygenation coefficient was 0.23 $days^{-1}$ in their study. However, the coefficient varies from

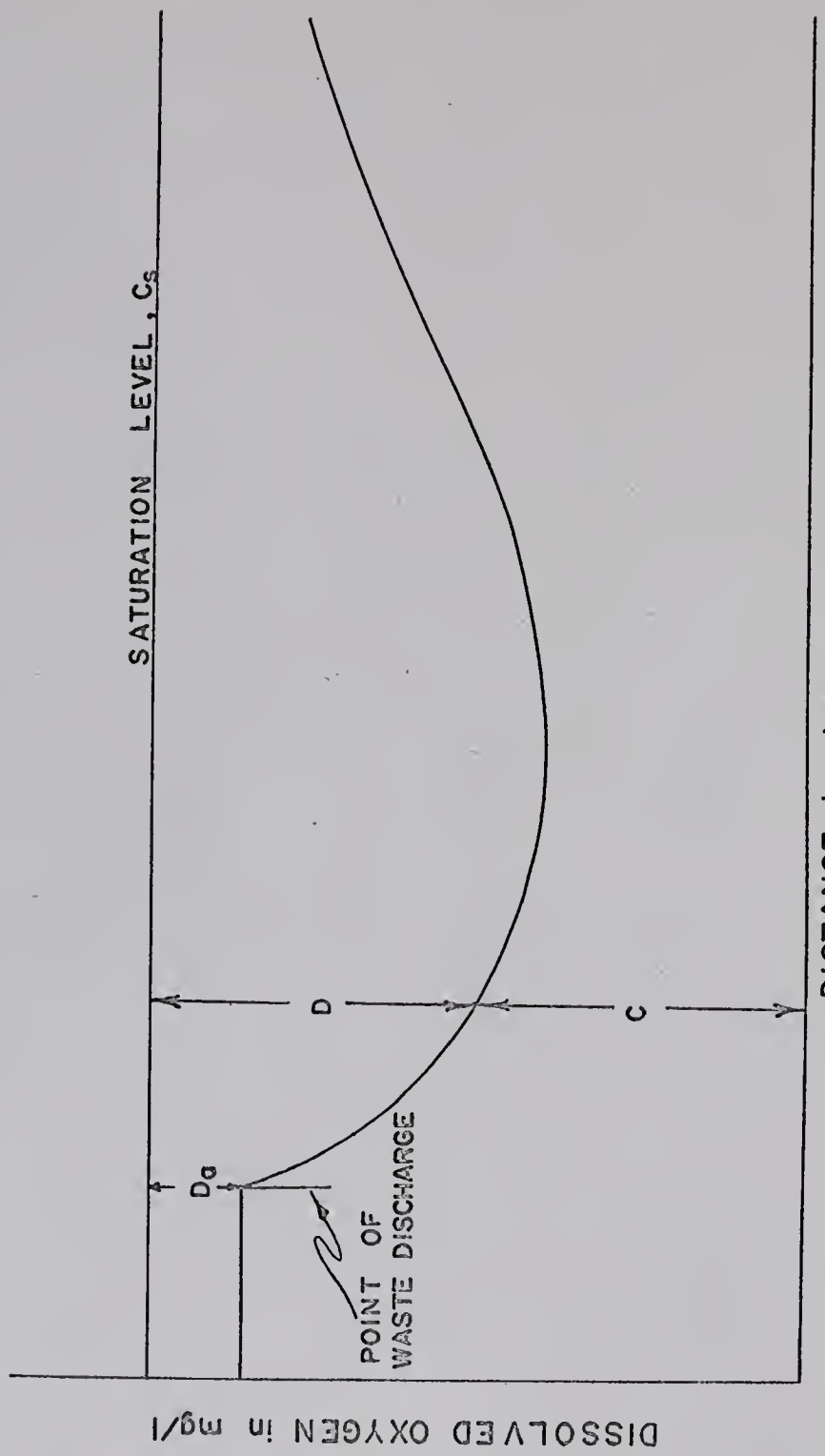


FIGURE 10. DISSOLVED OXYGEN SAG CURVE

river to river and for different wastes, and estimates have ranged from 0.1 to 0.7 days^{-1}. Because of the variation in K_1 and K_2, Churchill and Buckingham[47] used linear regression to develop a predictive equation for dissolved oxygen. They felt that the results of using K_1 and K_2 as constants for predicting dissolved oxygen under varying conditions were highly questionable. Kothandaraman[48] studied K_1 values calculated from data on the Ohio River and found them to be indistinguishable from a random sample. He also found the K_1 values to vary randomly according to Gaussian probability with a mean of 0.173 day^{-1} and a standard deviation of 0.066 day^{-1}.

Variation in the reaeration constant has been found in several investigations.[33, 49, 50] In each study, empirical expressions were derived to predict K_2 as a function of stream velocity and depth. Langbein and Durum[46] developed Equation (46) to predict K_2 over a wide range of data collected in the field and laboratory. They presented a graphical representation of the data points which showed considerable scatter of the points about Equation (46). Kothandaraman[48] examined data reported by the TVA[33] and performed a regression analysis to obtain an empirical equation for K_2 of the form:

$$K_2 = 5.827 \frac{v^{.924}}{d_m 1.705} \qquad \text{.....} \qquad (49)$$

He found that percentage variation about Equation (49) was normally distributed with a mean of zero and a standard deviation of 0.368.

In view of Kothandaraman's findings about the random behavior of K_1 and K_2, it is easier to understand the discrepancy between predicted and observed deficits. If K_1 and K_2 can be represented by a random

function, the deficit in Equation (49) is a function of random variables and has a probability distribution of its own. In this study, the dissolved oxygen deficit will be assumed to be a function of random components and thus will have a mean and standard deviation. This will permit a probabilistic expression for the effect of the waste load--for example, 100,000 lb BOD/day may cause a violation 5% of the time. This means that if the same stream flow and waste loads were combined on one hundred occasions, one would expect a violation of the stream standard on five occasions.

Additional Sources and Sinks of Dissolved Oxygen

Other investigators have attempted to add additional sources and sinks of dissolved oxygen to the Streeter-Phelps Equation. Modifications are warranted in situations where these additional sources and sinks play significant roles in the stream's dissolved oxygen balance. Thomas[51] presented a technique for stream analysis in which he added a rate constant K_3 to Equation (42) to represent the removal of organic material by sedimentation. This was the first modification to the Streeter-Phelps Equation. Other investigators have added parameters such as K_b to account for organic addition from runoff and scour, D_b for benthol demand, R for oxygen consumption by respiration of aquatic biota, and P for oxygen producted by photosynthesis. When these parameters are included in Equations (43) and (48), the result is a cumbersome equation for dissolved oxygen deficit and organic removal:

$$\frac{dL}{dt} = -(K_1 + K_3) L + K_b \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (50)$$

$$\frac{dD}{dt} = K_1 L - K_2 D - P + D_b + R \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (51)$$

An extensive data survey is necessary to estimate the values of

the parameters. It is often hard to obtain data from which reliable estimates of the parameter can be made. For example, Camp[52] found it necessary to assume $K_3 = 0.1$ in order to estimate K_1 from the collected data in a stream study. In his analysis, Camp found D_b to range from 0.5 to 1.0 mg/l/day; (P-R), 0.55 to 2.9 mg/l/day; and K_3, 0.07 to 0.11 day^{-1}.

In many instances, one cannot neglect the terms represented in the modification to the Streeter-Phelps, but, on the other hand, to try to determine the relative contribution from each term requires extensive data collection. Also, the variance in K_1 and K_2 must be taken into account when determining the other parameters. Because of the difficulty in estimating the parameters, Moreau and Pyatt[53] suggested lumping the added parameters to the Streeter-Phelps Equation into a composite term. These terms could be determined from field data by subtracting the contributions of K_1 and K_2 in the oxygen balance of the stream. Moreau and Pyatt represent the term in the organic load by r in Equation (42) and in the deficit equation by s:

$$\frac{dL}{dt} = -K_1 L + r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (52)$$

$$\frac{dD}{dt} = K_1 L - K_2 D + s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (53)$$

The signs of r and s terms are important. In Equation (52), r is positive, which represents added organic loading, and in Equation (53), s is positive, representing a dissolved oxygen sink. The combination of these positive terms, therefore, causes the predicted dissolved oxygen sag curve to be a conservative estimate. Equation (52) and (53) will be used in the derivation of the dissolved oxygen equation used in this study.

Dissolved Oxygen Equation Used in This Study

Since the stream temperature changes after discharge from the reservoir, this factor must be included in the determination of the dissolved oxygen saturation level. Liebman[54] proposes a method for handling the changing saturation level by assuming a linear relationship within reaches:

$$C_s(t) = C_{si} + Bt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (54)$$

where

$$B = \frac{C_{sf} - C_{si}}{T_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (55)$$

$C_s(t)$ is the saturation level at time t, C_{si} is the saturation level at the top of the reach, C_{sf} is the saturation level at the bottom of the reach, T_t is the travel time in the reach, and B is the slope of the saturation curve within the reach. This is a good approximation since the saturation level is almost linear with temperature. The slope B is positive when the water temperature drops within a reach and thus represents a dissolved oxygen sink. The final representation in differential form is:

$$\frac{dL}{dt} = - K_1 L + r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (56)$$

and

$$\frac{dD}{dt} = K_1 L - K_2 D - B + s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (57)$$

Integration of Equations (56) and (57) for steady, uniform flow yields:

$$L = (La - \frac{r}{K_1}) e^{-K_1 t} + \frac{r}{K_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (58)$$

and,

$$D = \frac{K_1 La - r}{K_2 - K_1} (e^{-K_1 t} - e^{-K_2 t}) + (Da - \frac{B}{K_2}) e^{-K_2 t} + \frac{r+s}{K_2} (1 - e^{-K_2 t})$$

$$\ldots\ldots\ldots\ldots \quad (59)$$

As pointed out by Kothandaraman,[48] the "constants" K_1 and K_2 can be represented by probability distribution functions each having its own mean and variance. Due to the uncertainty of r and s, Moreau and Pyatt[53] suggest that each be assumed to be random variables which may be represented by probability distribution functions. Therefore, in this study K_1, K_2, r and s are assumed to be random variables which may be represented by a probability distribution function having means μ_{K_1}, μ_{K_2}, μ_r, μ_s and variances $\sigma^2_{K_1}$, $\sigma^2_{K_2}$, σ^2_r, σ^2_s. One would expect that K_1 would be independent of K_2 since K_1 represents a biological process and K_2 is a physical process. For the same reason, K_1 is independent of r. However, K_1 and s represent biological processes which, for purposes of this analysis, will be assumed to be independent. It is felt that any dependence would contribute only a small degree of variance to the dissolved oxygen deficit. In a like manner, the remaining parameters are assumed independent. Therefore, the dissolved oxygen deficit becomes a function of independent random variables and has a probability distribution of its own.

Meyer[55] shows that a function Y of n independent random variables $x_1, x_2, \ldots, x_n$ (for instance, $Y = f(x_1\ x_2, \ldots, x_n)$) has the following approximations for its expected value and variance:

$$E(Y) \approx f(\mu_1, \mu_2, \ldots, \mu_n) + \frac{1}{2} \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2} \sigma_i^2 \quad \ldots\ldots \quad (60)$$

$$Var(Y) \approx \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_i^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (61)$$

where μ_i and σ_i^2 are the mean and variance of the random variable x_i.

Taking the indicated partial derivatives in Equations (60) and (61), the mean and variance of L and D are:

$$E(L) = L + \frac{1}{2}\left\{\frac{\partial^2 L}{\partial K_1^2}\sigma_{K_1}^2 + \frac{\partial^2 L}{\partial K_2^2}\sigma_{K_2}^2\right\} \quad \text{..... (62)}$$

$$\text{Var}(L) = \left(\frac{\partial L}{\partial K_1}\right)^2 \sigma_{K_1}^2 + \left(\frac{\partial L}{\partial r}\right)^2 \sigma_r^2 \quad \text{..... (63)}$$

$$E(D) = D + \frac{1}{2}\left\{\frac{\partial^2 D}{\partial K_1^2}\sigma_{K_1}^2 + \frac{\partial^2 D}{\partial K_2^2}\sigma_{K_2}^2 + \frac{\partial^2 D}{\partial r^2}\sigma_r^2 \frac{\partial^2 D}{\partial s^2}\sigma_s^2\right\} \quad \text{.. (64)}$$

$$\text{Var}(D) = \left(\frac{\partial D}{\partial K_1}\right)^2 \sigma_{K_1}^2 + \left(\frac{\partial D}{\partial K_2}\right)^2 \sigma_{K_2}^2 + \left(\frac{\partial D}{\partial r}\right)^2 \sigma_r^2 + \left(\frac{\partial D}{\partial s}\right)^2 \sigma_s^2 \quad \text{... (65)}$$

The partial derivatives of Equations (62), (63), (64) and (65) are given in Appendix A along with the expressions for E(L), Var(L), E(D) and Var(D). The results of Kothandaraman[48] indicate that the distribution function of the dissolved oxygen sag is approximately Gaussian. If such an assumption can be made, confidence limits of a predetermined probability can be established for the deficit when the expected value and variance are known. For instance, 90% of all values of a Gaussian distribution lie within the interval $\mu \pm 1.645\ \sigma$, where μ is the expected value and σ is the standard deviation (σ^2 = variance). In the case of dissolved oxygen, if the standard required that dissolved oxygen should remain above 4.0 mg/l 95% of the time, the standard would be met if the maximum deficit, calculated as $E(D) + 1.645\ \sqrt{\text{Var}(D)}$, did not cause the dissolved oxygen to fall below 4.0 mg/l. When the minimum dissolved oxygen is larger than 4.0 mg/l, a maximum allowable waste load can be determined which satisfies the standard.

Sensitivity Analysis

The expressions for the variance of L and D afford a convenient method for determining relative sensitivities to the parameters K_1, K_2, r, and s. The terms on the right hand side of Equation (65) are the contribution of the respective parameters to the total variance. By substituting nominal values of the parameters into Equation (65), one can evaluate the partial derivatives and determine which parameters cause the largest variance in D. Such an analysis was made for several values of K_1, K_2, r, and s. Kothandaraman found the deoxygenation coefficient for the Ohio River to have a coefficient of variation, C_v, of 0.38 ($C_v = \mu/\sigma$). If this value is assumed constant, for a K_1 of 0.23 $days^{-1}$, the standard deviation would be 0.0874 $days^{-1}$. This relationship was assumed to hold for K_1 in order to determine its standard deviation. For the reaeration coefficient, the standard deviation was assumed to be 0.368% of the mean, as found by Kothandaram. The remaining nominal values are given in Table 2. A time of one day was chosen to make the analysis since most river basins are subdivided into reaches with travel times of about one day.

Examination of the results in Table 2 reveals that the variance of D is caused entirely by K_1, K_2, and s; however, the expected value of D is dependent on all four parameters. The percentage of the variance from K_2 increases with increasing K_2 indicating that in swift, shallow streams variation in the deficit is due mostly to variation in K_2. As K_2 decreases the relative importance of K_1 in the variation of D increases. Therefore, for deep, sluggish streams a reliable estimate

TABLE 2

SENSITIVITY ANALYSIS

VALUES OF PARAMETERS						PERCENT VARIANCE FROM						PERCENT VARIANCE FROM	
K_1	K_2	r	s	E(D)	Var(D)	K_1	K_2	s	r	E(L)	Var(L)	K_1	r
.23	.6	1.0	1.0	3.49	.67	44	35	21	-	8.87	.73	73	27
.23	.6	-1.0	1.0	3.32	.59	39	37	24	-	7.08	.63	68	32
.23	.6	1.0	1.0	1.98	.54	54	20	26	-	8.87	.73	73	27
.23	.9	1.0	1.0	2.98	.65	34	49	17	-	8.87	.73	73	27
.23	.4	1.0	1.0	4.19	.68	58	14	28	-	8.87	.73	73	27
.4	.6	1.0	1.0	4.42	1.10	55	31	13	-	7.61	1.33	87	13
.23	.6	2.0	2.0	4.34	.79	42	40	18	-	9.76	.79	75	25
.23	.6	.5*	.5*	3.07	.51	54	38	7	-	8.42	.56	91	9
.23	.3	.5*	.5*	3.71	.50	74	16	9	-	8.42	.56	91	9
.4	.6	.5*	.5*	3.97	.90	63	33	4	-	7.19	1.14	96	4
.30	.2	.25*	.25*	4.19	.69	86	7	7	-	7.67	.78	94	6

D_a = 2.0 mg/1 t = 1.0 day σ_r = 0.5 $*\sigma_r$ = 0.25

L_a = 10.0 mg/1 B = 0.0 mg/1/day σ_s = 0.5 $*\sigma_s$ = 0.25

of K_1 would be more important than streams with high K_2 values. An increase in K_1 from 0.23 to 0.40 caused a large increase in the variance of D. Such increases in K_1 might occur during warm summer months, the period of most pollution problems.

Large changes in r and s cause little change in their contributions to the variation of D. When s is negative, corresponding to a photosynthetic oxygen source, the expected value of D decreases and the relative contribution of K_2 in the variance decreases. This indicates a decreasing importance in K_2 for streams with a photosynthetic oxygen source.

Also listed in Table 2 is the effect of K_1 and r on L. One can see that K_1 has considerably more contribution in the variance of L than r. Therefore, the estimation or determination of K_1 is more important than r. In summary, the sensitivity analysis reveals that most of the variation in D is caused by variation in K_1 and K_2 while in some cases s plays a more important role. It would seem, therefore, that in field studies more time and data collection should be devoted to determination of K_1 and K_2 than to r and s.

Application of Equations (63) and (65)

The application of Equations (63) and (65) requires special consideration if the equations are used in piecemeal fashion from reach to reach. In such cases the dissolved oxygen deficit and remaining BOD at the end of a reach are used as the initial deficit, D_a, and ultimate BOD, L_a, in the next reach. If D_a and L_a are deterministic values at the beginning of the first reach and take on the values of D and L at the beginning of the second reach, then D_a and L_a are no

longer deterministic, but are stochastic. Since the deficit then becomes a function of two more stochastic variables, the partial derivatives of D with respect to these variables must be included in the determination of the variance of D. Therefore, $(\frac{\partial L}{\partial L_a})^2 \sigma_{L_a}^2$ should be added to Equation (63) and $(\frac{\partial D}{\partial D_a})^2 \sigma_{D_a}^2 + (\frac{\partial D}{\partial L_a})^2 \sigma_{L_a}^2$ should be added to Equation (65). The values for $\sigma_{L_a}^2$ and $\sigma_{D_a}^2$ are simply the variance of L and D at the end of the previous reach. The expected values of L and D at the end of a reach are used for L_a and D_a in the next reach.

In this section, a stochastic model for dissolved oxygen has been formulated as a function of various parameters which, because of the uncertainty in assuming them constant, are taken to be independent normally distributed random variables. The use of the stream temperature and dissolved oxygen models for determining the allowable waste loading will be discussed in the following section.

Determination of Allowable Waste Loading

The thrust of many pollution studies is toward finding the maximum allowable waste load the stream can assimilate under various conditions of stream flow and stream temperature. To determine waste loadings there are several parameters in the stream and dissolved oxygen models which must be known. In this study, stream velocity and depth are parameters in the stream temperature model and are needed for determining the reaeration rate in the dissolved oxygen equation; the equilibrium temperature for the stream is fundamental to the stream temperature model; the temperature at the upper and lower ends of each reach determine the dissolved oxygen saturation slope and the values of the

deoxygenation and reaeration constants within the reach; and the maximum deficit determines the allowable waste discharge.

Although the stream's velocity and depth are important in the overall methodology, the model used to predict the relationship is of little significance as long as it predicts reasonable values for the particular stream being studied. In this study, velocity and depth will be in terms of stream flow:

$$V_m = \alpha_v Q^{B_v} \quad \text{..... (66)}$$

$$d_m = \alpha_h Q^{B_h} \quad \text{..... (67)}$$

where α_v, B_v, α_h, and B_h are constants in each reach, which are determined from observed data. The first step in determining the allowable waste load under given conditions is to calculate the mean velocity and mean depth for each reach. The flow in any reach is assumed steady and uniform. Tributary inflows to a reach occur at the upper end of the reach.

The second step is the evaluation of the meteorological data to determine the equilibrium temperature. (The procedure for calculating the equilibrium temperature was discussed in Chapter II.) The equilibrium temperature is assumed constant for all the downstream reaches; however, when geographical situations indicate this assumption to be false, another equilibrium temperature can be calculated for the remaining stream sections.

After determining the equilibrium temperature, one calculates the stream temperature profile by the stream temperature model. The deoxygenation and reaeration coefficients are calculated using the average stream temperature within the reach.

The final step is to determine the minimum dissolved oxygen for a given waste discharge. The sag curve is calculated at the end of each reach, with the remaining organic load and dissolved oxygen deficit becoming the initial conditions in the next reach. The minimum dissolved oxygen is found when the slope of the sag curve becomes positive. Tracing back to the beginning of the previous reach and calculating the dissolved oxygen at the third points of each of the last two reaches will approximately locate the minimum point if the change in slope is not too large. The minimum dissolved oxygen is tested against the allowable minimum and adjustments are made in the loading until the minimum is within 0.02 mg/l of the standard. The waste loading which satisfies the dissolved oxygen standard within 0.02 mg/l is the maximum allowable waste load for the given stream flow and reservoir release temperature. The program listing for the determination of allowable waste loading is included in Appendix B.

In the next chapter a discussion of the factors which influence a stream's waste assimilative capacity will be presented. The concept of a tradeoff between release flows and temperatures will be introduced, with hypothetical curves depicting the relationship between waste assimilative capacity and the flow and temperature parameters.

IV. THE TRADEOFF BETWEEN STREAM TEMPERATURE AND STREAM FLOW

As mentioned in Chapter II, few investigators consider discharge temperature variation from reservoirs in water quality models. However, the variation in temperature may be quite significant and provides the basis for considering a tradeoff between the temperature of the release and the quantity of discharge. The term "tradeoff" in this study will mean that water at one temperature and discharge rate may be exchanged for water at a different temperature and discharge rate while still maintaining specified water quality standards downstream. The exchange is possible because of the variation in temperature within the reservoir.

The Effects of Temperature on Waste Assimilative Capacity

The effect of temperature on dissolved oxygen in streams is two-fold. First, temperature has a pronounced effect on the biological activity in the stream, and secondly, the physical parameters of the stream vary with stream temperature. In general, biological activity increases with increasing temperature. This phenomenon is caused by the dependence of enzyme reaction rates on temperature. Increasing temperatures cause higher reaction rates, which in turn stimulate metabolic activity and cause the oxygen requirements of the organism to increase. Thus, increasing the temperature in a stream causes faster oxidation of soluble organic material and a sharper rate

of decrease in the oxygen sag curve. The variation of the deoxygenation coefficient with temperature is often expressed as:[57]

$$K_{1,\,T} = K_{1,\,20}\, e^{.046(T-20)} \quad \text{..... (68)}$$

where $K_{1,\,20}$ is the value of the coefficient at 20°C; T is the temperature in degrees centigrade; and $K_{1,\,T}$ is the value of the coefficient at temperature T.

The physical properties affected by temperature relate primarily to dissolved oxygen resources in the stream. Increasing temperature causes the solubility of oxygen in water to decrease. This lowers the saturation level of dissolved oxygen (Equation (30)). Also, increasing temperature decreases the viscosity of water, the rate of molecular vibration, and the rate of oxygen dispersion. These factors cause the reaeration coefficient to increase with increasing temperature. The reaeration rate as a function of temperature is:[33]

$$K_{2,\,T} = K_{2,\,20}\, e^{.016(T-20)} \quad \text{..... (69)}$$

where $K_{2,\,20}$ is the reaeration coefficient at 20°C; T is the temperature in degrees centigrade; and $K_{2,\,T}$ is the reaeration coefficient at temperature T.

The high summer temperatures in streams cause lower levels of dissolved oxygen saturation and higher coefficients of reaeration and deoxygenation. However, comparing Equations (68) and (69) reveals that changing temperatures cause larger changes in K_1 than in K_2. A change of 5°C from 20°C causes a 20% change in K_1 and a 9% change in K_2.

When the biological and physical effects are combined at high temperatures, one observes that the demand on the stream's dissolved oxygen resources is highest at the time when the total available oxygen is lowest. The net result is a decrease in the stream's waste assimilative capacity during the summer months. If cool water were available in a reservoir and if it were of suitable quality, then releasing the cool water would increase the assimilative capacity. However, the degree of increase in assimilative capacity will depend upon the distance of the waste outfall from the reservoir. Since the cool water will warm as it passes through the basin, waste discharges far downstream will not have the benefit of the cool water.

Examples of the effects of temperature on the waste assimilative capacity were reported by Worley[44] and Krenkel [56] et al. Worley was investigating low flow augmentation requirements in the Willamette River Basin. He found that increasing the temperature from 20°C to 25°C caused a 14% decrease in the total allowable waste load in the basin for a streamflow of 5500 cfs. Krenkel et al. studied the effects of impoundment on the Coosa River in Georgia. Under free flow conditions of 940 cfs, they reported large variations of allowable waste loading with temperature. Decreasing the temperature from 20°C to 15°C increased the allowable loading 40% from 45,000 lbs BOD/day to 63,000 lbs BOD/day, while a 5°C increase to 25°C decreased the allowable loading 20% to 32,000 lbs BOD/day. Such evidence supports the tradeoff among temperatures for the purpose of improving waste assimilative capacity.

The Effects of Stream Flow on Waste Assimilative Capacity

Streamflow influences the extent of dilution of the organic waste and the reaeration coefficient of the stream. By increasing flows, the concentration of the waste is decreased, and one would expect a corresponding increase in the assimilative capacity of the stream. Discussion will be presented in this section which questions the validity of this assumption.

The equation for determining the reaeration coefficient was given in Equation (46) as:

$$K_2 = 3.3 \frac{v}{d_m^{1.33}} \quad \text{.....} \quad (70)$$

Increasing flow increases the velocity of the stream as well as its depth, but the reaeration coefficient is influenced to a greater extent by the depth of the stream than by the velocity. Langbein and Durum[46] point out that the reaeration coefficient at any location decreases at about the 0.13 power of the discharge. Therefore, increasing flows causes a decrease in the capacity of the stream to reaerate.

Low-flow augmentation as a means of water quality control is usually employed in the summer months. If earlier releases at the reservoir have emptied the colder water, the releases will be at warmer temperatures, and by the time the flow reaches the waste discharge point the temperature will probably have reached equilibrium. The warmer temperature will cause high K_1 values in the stream. The sensitivity analysis in Chapter III indicated considerable variation in the dissolved oxygen deficit for high K_1 values and marked increases in the expected value of the deficit. The sensitivity analysis also

revealed larger variations in the deficit with decreasing K_2 values, as well as an increase in the expected value of the deficit. In view of these results, augmenting low flows with warm water releases would cause an increase in K_1 and a decrease in K_2 with correspondingly larger variations in the dissolved oxygen deficit. Therefore, considerable case study should be made before low-flow augmentation is used as a method for water quality control. Arbitrarily releasing water without regard to temperature and influences on K_2 may create water quality problems instead of solving them.

The Stream Flow Stream Temperature Tradeoff

In the previous sections, the effects of temperature and flow on waste assimilative capacity were discussed. With this background, it is possible to infer the existence of a tradeoff between water temperature and stream flow.

The effects of increasing temperature were said to decrease the stream's assimilative capacity. With this in mind, hypothetical curves for assimilative capacity versus temperature are shown in Figure 11a. The curves are shown for various flow rates. For flow rate Q_1, the assimilative capacity decreases with increasing release temperature. For the same temperature, increasing the flow increases the assimilative capacity, but the effect of this increase is shown to decrease as the curves slope to the right. The tradeoff can be seen by noticing that the assimilative capacity for the coldest discharge at flow rate Q_1 exceeds the assimilative capacity at flow rate Q_3. If the waste discharge were at a distance from the reservoir such that temperature had no effect on K_1, then the curves would become horizontal lines as shown in Figure 11a.

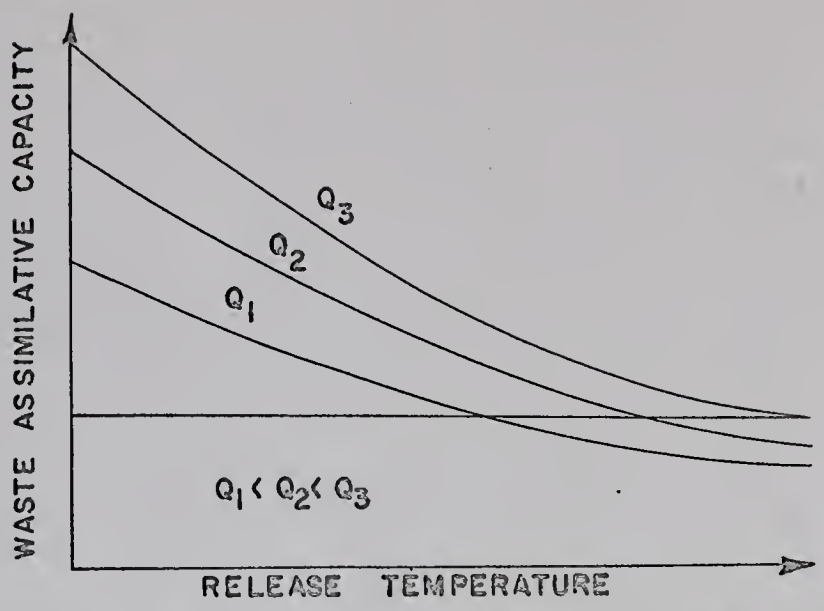


FIGURE a. WASTE ASSIMILATIVE CAPACITY vs. TEMPERATURE

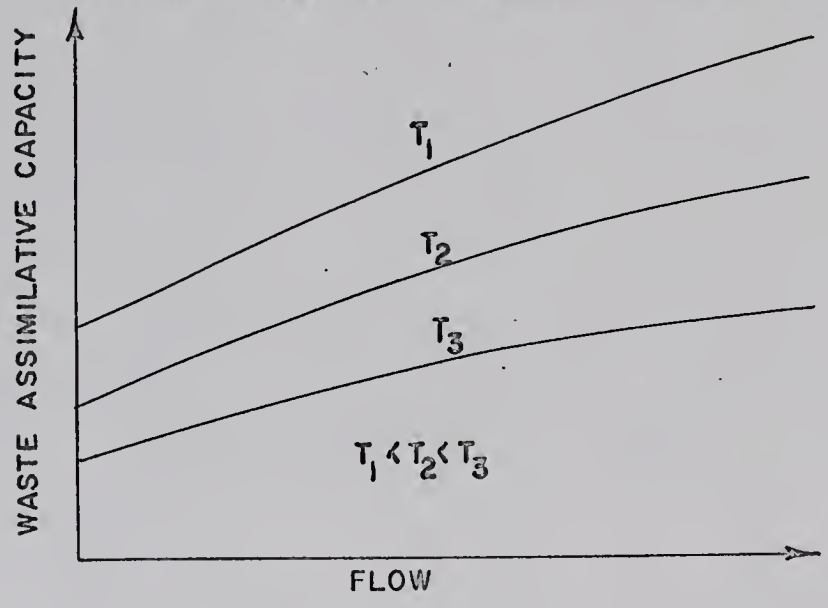


FIGURE b. WASTE ASSIMILATIVE CAPACITY vs. FLOW

FIGURE 11. HYPOTHETICAL CURVES
for WASTE ASSIMILATIVE CAPACITY
vs. RELEASE TEMPERATURE and FLOW

The tradeoff is illustrated, also, in Figure 11b. For the warmest temperature, T_3, the flow requirements become increasingly large for small increases in waste assimilative capacity. This is based on the assumption of diminishing returns for increasing flows. If no tradeoff exists, the curves of Figure 11b would collapse to the bottom curve.

For situations in which the tradeoff in temperature does not exist, the water quality variations in the reservoir have very little effect on the stream's assimilative capacity. The only alternatives to meeting the stream standard become waste treatment, with methods such as variable waste treatment, in-stream aeration and increasing flows. Since the methodology developed in this dissertation is based on the consideration of reservoir water quality variations, the existence of a tradeoff is required for application of the methodology. In Chapter V, earlier water quality models will be discussed and the strategy employed in this study will be developed.

V. DEVELOPMENT OF METHODOLOGY

The construction of a reservoir in a river basin causes water quality variation within the reservoir, which in turn modifies the annual variation in the downstream reaches. Thermal stratification in the reservoir presents the opportunity for selectively withdrawing the quality of water which best meets downstream requirements. In this chapter, a methodology will be described which enables the selection of the "best" water at successive time intervals. The methodology is based upon the concept of a tradeoff among the layers within the reservoir.

Description of the Problem

The river-reservoir system is shown in Figure 12. The system consists of a reservoir upstream from a large municipality, which discharges its wastes into the stream. Water quality standards are established for the reaches downstream from the reservoir. There are various alternative methods at the two control points, reservoir and waste treatment plant, that can be employed to meet the water quality standards. The objective is to determine which alternative or combination of alternatives is the most economical. This can be achieved by comparing the costs of various schemes.

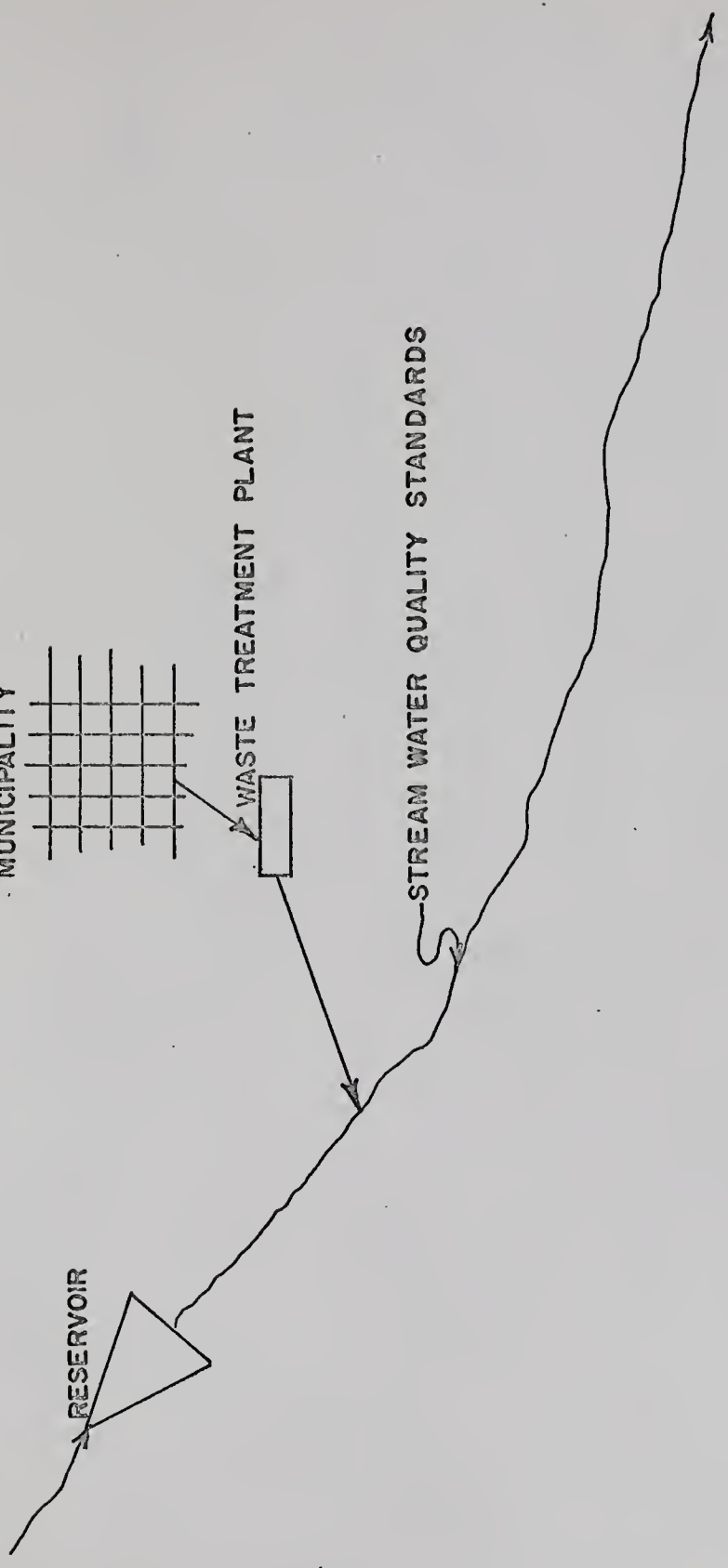


FIGURE 12. RIVER–RESERVOIR SYSTEM

Given any combination of alternatives (e.g., reservoir mixing and waste treatment) and various constraints at the reservoir (such as minimum releases for navigation and maximum flood control storage), the problem is to minimize annual downstream waste treatment costs while maintaining water quality standards and satisfying the constraints. It is not sufficient to minimize downstream treatment cost during each time interval because this policy would release the best quality each month with no consideration to the following months. Conceivably, this might lead to the point that all water releases violate the stream standards. Nor is it sufficient to release the minimum quality of water which, when combined with waste treatment, meets the stream standards. A policy of this nature conserves the higher quality water in the reservoir instead of allocating the water among the various time intervals. This policy also would encourage high levels of BOD removal in the waste discharge and implies that even during severe water quality periods the high quality water might remain in the reservoir. The best policy is to allocate the water in the reservoir in such a manner that annual downstream waste treatment is minimized. Since the reservoir is thermally stratified during the heating cycle, and most water quality problems occur during the summer and early fall, the term "annual" will apply to the period from April to October. Most reservoirs are only slightly stratified between November and March, with the result that there is no temperature tradeoff within the reservoir.

To minimize annual waste treatment costs, an optimization technique is needed which will enable the consideration of the tradeoff

between release temperature and flow. In each time period, decisions must be made on the temperature of the release water and the amount of water to release. Since the decisions made at each time interval affect the state of the reservoir in the next time interval, the technique must be able to incorporate the dynamic characteristics of the reservoir. Before formulating the optimization technique, discussion will be presented on the different techniques used in water quality studies.

Water Quality Optimization Models

There are several features which all water quality optimization models have in common. First, the models must be formulated so that decisions can be translated into a ranking function or objective function. The ranking function serves as a means for determining the effectiveness of decisions, and should provide a well-defined ordering of alternative decisions. The optimization technique is applied to this function.

Second, the decisions are required to meet certain constraints, such as maintaining water quality standards or releasing of a minimum quantity of water from the reservoir at all times. Sometimes the constraints are not imposed directly on the decision, but rather on the consequences of the decision. For example, if a decision were made on the quantity of waste discharge at a treatment plant and the constraint was a minimum dissolved oxygen level in the stream, the decision would be disallowed if the response of the stream to the waste violated the standard.

A third feature is the result of the constraints. Most water

quality models are formulated to investigate the effects of pollution on the dissolved oxygen standard. Therefore, a predictive model for the dissolved oxygen response to various waste loadings is necessary. The model selected is usually the Streeter-Phelps Equation.

Fourth, an optimization technique is chosen which best handles the formulated model. The technique should be amenable to programming on a digital computer. In water quality studies, the techniques fall into three categories: nonlinear programming, linear programming, and dynamic programming. An example of the application of each of the techniques will be presented.

Kerri [58] developed a model which might be used by regulatory agencies to determine waste treatment requirements for several municipalities and industries within a river basin. His objective function was to minimize total waste treatment costs in the basin for various dissolved oxygen standards. The degree of treatment required at each treatment plant was determined at the optimal solution. He used nonlinear programming to solve for the degree of treatment and the Streeter-Phelps Equation for predicting the dissolved oxygen profile in the river. The model was tested on the Willamette river basin for a critical low flow period to investigate several treatment alternatives. The model provides a means for determining the economic impact of setting uniform waste treatment standards. Kerri found that in the Willamette basin, the minimum annual cost solution was $2,999,000 for a dissolved oxygen standard of 5.0 mg/1. At the optimal solution, the municipalities were not required to provide waste treatment, but four sulfite pulp mills were required to maintain 90%

BOD removal. By forcing all municipalities and industries to provide 74% BOD removal, the cost increased to $4,733,000.

Liebman[54] and ReVelle et al.[59] applied different optimizing techniques to the same problem described by Kerri. The objective function was that of minimizing total basin costs of waste treatment. Liebman formulated the model as minimizing $\sum_{i=1}^{N} c_i(p_i)\ldots$ subject to $o_i^m \geq s_i$ $(i=1,2,3,\ldots,N)$: where p_i is the percentage BOD removal at the i^{th} discharge point; o_i^m is the minimum dissolved oxygen concentration in reach i resulting from the i^{th} discharge of waste and all preceding waste discharges; s_i is the minimum allowable dissolved oxygen concentration in reach i. The function $c_i(p_i)$ is the total annual cost of providing p_i percentage BOD removal at the i^{th} treatment plant. The function can be expressed in mathematical form or in tabular form. Liebman used the Streeter-Phelps Equation to predict the dissolved oxygen response in the stream. He used dynamic programming to optimally allocate the dissolved oxygen resources in the stream. The model was tested on the Willamette River Basin and yielded a minimum cost similar to that of Kerri; however, in Liebman's study all municipalities were required to furnish primary treatment of 35% BOD removal. In Liebman's model, the effects of changing the dissolved oxygen standard in the downstream direction can be studied permitting monetary evaluation of the standards in each reach.

ReVelle et al. assumed a linear relationship between organic loading and dissolved oxygen response. They required the cost function to be linear and minimized the total waste treatment cost in a

basin subject to the inequality constraints of the water quality standards (e.g., dissolved oxygen required to be greater than or equal to the minimum allowable dissolved oxygen) in each reach. The Streeter-Phelps Equation was linearized and formulated as equality constraints in each reach. They used linear programming to solve for the optimal solution. When solving the linear programming algorithm, the reach in which the water quality constraint is restrictive can be found in the solution to the dual. For each water quality constraint, a dual variable exists which will be non-zero if the constraint is binding at the optimum. By relaxing that constraint, the minimum cost can be decreased. ReVelle et al. applied the linear programming technique to the Willamette River Basin and achieved approximately the same results as Liebman; however, they were able to identify the reach in which water quality standards could be lowered to reduce the overall treatment costs.

The water quality models described above may aid the regulatory agency in setting stream standards. In each model, the objective is to lower waste treatment costs for the entire basin. The optimal efficiency is determined for each plant to satisfy the standards during the low-flow period. The low flow usually selected for sewage treatment plant design is the lowest consecutive seven-day or ten-day flow which has a frequency of once in ten years. To construct a plant based on this low-flow criterion means that the plant will actually "over treat" for the stream a large part of the time. Moreau [10] suggests the use of a steady-state treatment plant and an auxiliary unit which is used during low-flow periods for providing

sufficient BOD removal to meet stream standards. He showed that the combined cost of the steady-state and auxiliary units was less than the cost of a single unit designed to satisfy the low-flow criterion.

Davis [5] investigated the use of advanced waste treatment during critical flow periods as a substitute for low-flow augmentation. He found that in the Potomac Estuary advanced waste treatment was approximately the same cost as low flow augmentation. Advanced waste treatment provided more flexibility since the processes could be operated only during the critical period. He also showed that operating costs can be evaluated vis a vis capital costs in the selection of an advanced waste treatment process. In his example, step aeration plus microstraining had a capital cost of $20,000,000 and an annual operating cost of $56,000 while chemical precipitation had a capital cost of $200,000 and an annual operating cost of $426,000. If the two processes are compared at an annual operation of 3.5 months, a 50 year project life and a 4% interest rate, the present value of treatment costs are $38,700,000 for the step aeration plus microstraining and $32,300,000 for chemical precipitation. At higher interest rates chemical precipitation becomes more favorable.

For many municipalities, the establishment of stream standards will require the construction of additional treatment plants. In view of the work of Moreau and Davis, the addition of a low capital cost, high operating cost auxiliary unit may be the most economical alternative. If auxiliary units were installed, the objective could be changed to minimizing the operating cost of the auxiliary treatment unit.

Little [60] formulated a model for optimizing reservoir releases for hydroelectric power generation, which is similar to minimizing auxiliary treatment costs. Since future river flows are unknown and the release decision is made under uncertainty, he chose to maximize the expected return, not the return itself. Little considered the case of a hydroelectric dam and a steam plant operating conjunctively to produce electric power for a region. The steam plant was used to supply the demands which water could not satisfy. His objective function was minimizing expected supplemental costs from the steam-electric plant. He used stochastic dynamic programming to determine the optimum releases for minimizing the objective function. He assumed that the inflows to the reservoir were random variables which could be represented by a Markov model of lag one. The optimization technique required that the problem have a fixed end condition from which to start. The expected value of the cost function was then computed using the probability distribution of inflows for each time interval beginning at the known point and working back to the present. This technique is called backward looking stochastic dynamic programming.

Young [61] commented on the various dynamic programming techniques and developed what is termed Monte Carlo Dynamic Programming. Young used a forward looking dynamic programming model to determine reservoir operating rules in flood control reservoirs. He assumed the annual inflow to be known and then optimized the release quantity. By generating many years of synthetic flows and optimizing the draft for every additional flow, Young could find the optimal release rule. Young also developed another method of determining the release rule.

By assuming the optimal draft to be a linear function of storage, inflow, and an inflow forecast, the optimal draft can be determined for each time interval by making a regression analysis on the selected variables.

In reviewing the previous models, one observes that dynamic programming was used for optimizing releases from reservoirs. This technique is particularly useful for nonlinear cost functions, for problems involving two or more decision variables, and for a system which changes with time. The problem of interest in this study is that of selecting the release water temperature and flow which minimizes downstream treatment costs during the summer months. Since this involves two decision variables and a reservoir which changes in accordance with the decisions, dynamic programming will be selected as the optimizing technique. The formulation of the algorithm will be presented in the next section.

Dynamic Programming

There are several general features of dynamic programming which make the procedure applicable to water resource studies. The technique is especially useful in determining the allocation of a limited resource among various uses. In the problem at hand, it is necessary to allocate water from a reservoir for succeeding time intervals. Dynamic programming is very useful for multiple decision problems which change over time in accordance with decisions made in earlier time intervals. The dynamic nature of a reservoir is apparent when one considers the marked effect releasing water from the various layers has on the state of the reservoir. Another important feature

of dynamic programming is that nonlinear cost functions can be used. This feature is particularly useful if the cost function itself changes with time. If several constraints are required in the formulation of a dynamic programming algorithm, the constraints tend to simplify the problem instead of making it more complex. This point will be brought out in the development of the algorithm.

To facilitate the derivation of the algorithm, a one-dimensional decision process will be considered. It is desired to allocate a known quantity, Q, of water over several time periods, N. The water is to be allocated in such a manner that the total cost, C, of auxiliary waste treatment is minimized over the intervals. The cost, c_i, in the i^{th} period is some function of the water, q_i, released in that period:

$$c_i = g_i(q_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (71)$$

The objective function is

$$\text{minimize } C(q_1,q_2,\ldots q_N) = \sum_{i=1}^{N} c_i = \sum_{i=1}^{N} g_i(q_i) \quad \ldots\ldots \quad (72)$$

The minimization is subject to the constraint that the sum of the q_i's must be less than or equal to the total resource Q. This is expressed as:

$$\sum_{i=1}^{N} q_i \leq Q \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (73)$$

The sequence of functions $f_N(q)$ is introduced as follows:

$$f_N(q) = \min_{\{q_i\}} C(q_1,q_2,\ldots,q_N) \quad \ldots\ldots\ldots \quad (74)$$

The function $f_N(q)$ is the optimal solution for an allocation of the quantity Q of resource to N time intervals. It is seen that when N

is 1, $f_N(q)$ is simply the minimum of Equation (71) for i=1; i.e.,

$$f_1(q) = g_1(q) \tag{75}$$

To develop the dynamic programming algorithm, it is necessary to find a recurrence relationship connecting $f_N(q)$ with $f_{N-1}(q)$ for an arbitrary N and resource Q. If q_N is the amount of resource allocated to period N, then, regardless of the cost of q_N, the remaining resource $Q-q_N$ is used to obtain the minimum cost in the remaining N-1 periods. By definition, the minimum cost of allocating $Q-q_N$ resource to N-1 periods is the function $f_{N-1}(Q-q_N)$. An initial allocation of q_N to period N has the cost:

$$c_N = g_N(q_N) + f_{N-1}(Q-q_N) \quad \cdots\cdots\cdots \tag{76}$$

The optimal allocation of q_N resource to the N^{th} period is that allocation which minimizes Equation (76). The basic functional equation can be expressed as:

$$f_N(Q) = \min_{0 \le q_N \le Q} \{g_N(q_N)+f_{N-1}(Q-q_N)\} \ \ldots \tag{77}$$

In the derivation of Equation (77), a general principle was used: The Principle of Optimality. This principle expressed by Bellman [62] is: "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

To make the allocation of the resource Q amenable to digital programming, the resource is divided into discrete units. Also, to make the problem realistic, constraints are placed on the possible states of Q. Such constraints might be a minimum release, q_{min}, which must be maintained at all times for navigation, and a maximum release,

q_{max}, which might be the physical limit of the reservoir release structures. These constraints change Equation (77) into the form:

$$f_N(Q) = \min_{q_{min} \leq q_N \leq q_{max}} \{g_N(q_N) + f_{N-1}(Q-q_N)\} \quad \ldots\ldots\ldots\ldots \quad (78)$$

Example Problem

To illustrate the use of the algorithm, an example allocation problem is hypothesized. It is desired to allocate ten units of water over four time periods in such a manner as to minimize the total cost of treating waste to maintain water quality standards. A minimum discharge of one unit must be maintained during each interval, and the maximum discharge is four units. The cost of waste treatment as a function of units of discharge is given in Table 3.

TABLE 3

COST MATRIX FOR DYNAMIC PROGRAMMING EXAMPLE

	Costs for Each Period			
Units Released	1	2	3	4
1	3	4	5	7
2	2	2	3	4
3	0	1	1	1
4	0	0	0	0

Costs for waste treatment during the first time interval are given by Equation (75). These values are simply the costs as given for period one in Table 3. For period two, the function $f_2(Q)$ is developed by finding the minimum cost of allocating from two to eight units to the first two periods. A minimum of two units in the result of the constraint that one unit must be released at all times and

eight units is the result of allocating a maximum of four units to each of the first two intervals. The function $f_2(Q)$ is given in Table 4. To illustrate the formation of Table 4 with Q equal to five units, there are four possible ways to allocate the resource between periods one and two: one unit in period two and four units in period one; two units in two and three units in one; three units in two and two in one; or, four units in two and one unit in one. The fourth column of Table 4 gives the costs of these possibilities. It is seen that the optimum allocation of five units to periods one and two is two units in period two and three units in period one. Thus $f_2(5)$ equals two is the value retained in the last column of Table 4.

For period three, Table 5 is formed in the same manner using only the values for $f_2(Q)$. During this period, the values of Q may range from three to nine. Nine is the maximum since one unit must be saved for release during the fourth interval. Only the feasible possibilities are included in Table 5. The allocation of four to ten units is given in Table 6. Examination of the function $f_4(Q)$ reveals that the minimum cost occurs when Q is ten. Beginning at this point and tracing back through Tables 6, 7, and 3, one can obtain the optimal set of decisions. From Table 6, the optimal decision is to allocate three units to period four and seven units to the first three periods. The optimal allocation of seven units in period three is obtained from $f_3(7)$ in Table 5. There are three possible optima for allocating seven units to periods one, two, and three. Such cases as this cause more than one optimal set of decisions.

TABLE 4

EVALUATION OF EQUATION (78)
FOR N EQUAL TWO IN EXAMPLE

Q	$g_2(q_2)$	$f_1(Q-q_2)$	$g_2(q_2)+f_1(Q-q_2)$	$f_2(Q)$
2	$g_2(1)$	$f_1(1)$	4 + 3 = 7	7
3	$g_2(1)$	$f_1(2)$	4 + 2 = 6	
	$g_2(2)$	$f_1(1)$	2 + 3 = 5	5
4	$g_2(1)$	$f_1(3)$	4 + 0 = 4	4
	$g_2(2)$	$f_1(2)$	2 + 2 = 4	
	$g_2(3)$	$f_1(1)$	1 + 3 = 4	
5	$g_2(1)$	$f_1(4)$	4 + 0 = 4	
	$g_2(2)$	$f_1(3)$	2 + 0 = 2	2
	$g_2(3)$	$f_1(2)$	1 + 2 = 3	
	$g_2(4)$	$f_1(1)$	0 + 3 = 3	
6	$g_2(1)$	$f_1(5)$*	4 + * = *	
	$g_2(2)$	$f_1(4)$	2 + 0 = 2	
	$g_2(3)$	$f_1(3)$	1 + 0 = 1	1
	$g_2(4)$	$f_1(2)$	0 + 2 = 2	
	$g_2(5)$*	$f_1(1)$	* + 3 = *	
7	$g_2(1)$	$f_1(6)$*	4 + * = *	
	$g_2(2)$	$f_1(5)$*	2 + * = *	
	$g_2(3)$	$f_1(4)$	1 + 0 = 1	
	$g_2(4)$	$f_1(3)$	0 + 0 = 0	0
	$g_2(5)$*	$f_1(2)$	* + 2 = *	
	$g_2(6)$*	$f_1(1)$	* + 3 = *	
8	$g_2(1)$	$f_1(7)$*	4 + * = *	
	$g_2(2)$	$f_1(6)$*	2 + * = *	
	$g_2(3)$	$f_1(5)$*	1 + * = *	
	$g_2(4)$	$f_1(4)$	0 + 0 = 0	0
	$g_2(5)$*	$f_1(3)$	* + 0 = *	
	$g_2(6)$*	$f_1(2)$	* + 2 = *	
	$g_2(7)$*	$f_1(1)$	* + 3 = *	

*Violates constraint; therefore not feasible.

TABLE 5

EVALUATION OF EQUATION (78)
FOR N EQUAL THREE IN EXAMPLE

Q	$g_3(q_3)$	$f_2(Q-q_3)$	$g_3(q_3)+f_2(Q-q_3)$	$f_3(Q)$
3	$g_3(1)$	$f_2(2)$	5 + 7 = 12	12
4	$g_3(1)$	$f_2(3)$	5 + 5 = 10	10
	$g_3(2)$	$f_2(2)$	3 + 7 = 10	
5	$g_3(1)$	$f_2(4)$	5 + 4 = 9	
	$g_3(2)$	$f_2(3)$	3 + 5 = 8	8
	$g_3(3)$	$f_2(2)$	1 + 7 = 8	
6	$g_3(1)$	$f_2(5)$	5 + 2 = 7	
	$g_3(2)$	$f_2(4)$	3 + 4 = 7	
	$g_3(3)$	$f_2(3)$	1 + 5 = 6	6
	$g_3(4)$	$f_2(2)$	0 + 7 = 7	
7	$g_3(1)$	$f_2(6)$	5 + 1 = 6	
	$g_3(2)$	$f_2(5)$	3 + 2 = 5	5
	$g_3(3)$	$f_2(4)$	1 + 4 = 5	
	$g_3(4)$	$f_2(3)$	0 + 5 = 5	
8	$g_3(1)$	$f_2(7)$	5 + 0 = 5	
	$g_3(2)$	$f_2(6)$	3 + 1 = 4	
	$g_3(3)$	$f_2(5)$	1 + 2 = 3	3
	$g_3(4)$	$f_2(4)$	0 + 4 = 4	
9	$g_3(1)$	$f_2(8)$	5 + 0 = 5	
	$g_3(2)$	$f_2(7)$	3 + 0 = 3	
	$g_3(3)$	$f_2(6)$	1 + 1 = 2	2
	$g_3(4)$	$f_2(5)$	0 + 2 = 2	

TABLE 6

EVALUATION OF EQUATION (78)
FOR N EQUAL FOUR IN EXAMPLE

Q	$g_4(q_4)$	$f_3(Q-q_4)$	$g_4(q_4)+f_3(Q-q_4)$	$f_4(Q)$
4	$g_4(1)$	$f_3(3)$	7 + 12 = 19	19
5	$g_4(1)$	$f_3(4)$	7 + 10 = 17	
	$g_4(2)$	$f_3(3)$	4 + 12 = 16	16
6	$g_4(1)$	$f_3(5)$	7 + 8 = 15	
	$g_4(2)$	$f_3(4)$	4 + 10 = 14	
	$g_4(3)$	$f_3(3)$	1 + 12 = 13	13
7	$g_4(1)$	$f_3(6)$	7 + 6 = 13	
	$g_4(2)$	$f_3(5)$	4 + 8 = 12	
	$g_4(3)$	$f_3(4)$	1 + 10 = 11	11
	$g_4(4)$	$f_3(3)$	0 + 12 = 12	
8	$g_4(1)$	$f_3(7)$	7 + 5 = 12	
	$g_4(2)$	$f_3(6)$	4 + 6 = 10	
	$g_4(3)$	$f_3(5)$	1 + 8 = 9	9
	$g_4(4)$	$f_3(4)$	0 + 10 = 10	
9	$g_4(1)$	$f_3(8)$	7 + 3 = 10	
	$g_4(2)$	$f_3(7)$	4 + 5 = 9	
	$g_4(3)$	$f_3(6)$	1 + 6 = 7	7
	$g_4(4)$	$f_3(5)$	0 + 8 = 8	
10	$g_4(1)$	$f_3(9)$	7 + 2 = 9	
	$g_4(2)$	$f_3(8)$	4 + 3 = 7	
	$g_4(4)$	$f_3(7)$	1 + 5 = 6	6
	$g_4(4)$	$f_3(6)$	0 + 6 = 6	

If the first possible optimum is taken, then the decision in period three would be to allocate two units in period three and five units to periods one and two. From Table 4, it is seen that $f_2(5)$ is obtained by allocating two units in period two and three units in period one. This example illustrates the procedure of dynamic programming: start at a known point, such as $f_1(q)$ equals $g_1(q)$, and allocate the resource at each interval or stage; at the N^{th} stage the minimum cost is obtained and that resource allocated to the N^{th} period is the starting point for tracing backwards through the stages to obtain the optimal set of decisions.

At first, dynamic programming appears to be simply an enumeration of all possibilities of allocating the resource. In the example above, enumeration of all possibilities would result in determination of the costs for 4^4 possibilities. By using dynamic programming, the problem of finding one equation in four unknowns was decomposed into finding four equations with two unknowns. This resulted in evaluation of sixty combinations of two variables.

Two-Dimensional Algorithm

Since this dissertation involves determining from which layer the water should be released as well as the quantity to be released, it is necessary to use a two-dimensional algorithm for determination of the optimum decisions. This requires that the costs in each time interval be a function of not only q_i but also the layer released from, $\ell_{i,j}$. This is expressed as

$$c_i = q_i(q_i, \ell_{i,j}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (79)$$

where i is the period under consideration and j is the release layer

in the i^{th} period. Since the reservoir stratifies into three layers, j has the values 1, 2, or 3. The objective function is to

$$\text{minimize } C(q_1, q_2, \ldots, q_N; \ell_{1,j}, \ell_{2,j}, \ldots, \ell_{N,j}) = \sum_{i=1}^{N} c_i = \sum_{i=1}^{N} g_i(q_i, \ell_{i,j}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (80)$$

Proceeding as in the case of a one-dimensional allocation process, the function $f_N(q, \ell)$ is:

$$f_N(q, \ell) = \min_{\{q_i \atop \ell_{i,j}\}} C(q_1, q_2, \ldots q_N; \ell_{1,j}, \ell_{2,j}, \ldots \ell_{N,j}) \ldots (81)$$

The function $f_N(q, \ell)$ is now the minimum cost from the allocation of resource Q from layer ℓ over the N time periods. The recurrence relation is:

$$f_N(q,\ell) = \min_{q_{min} \le q \le q_{max}} \min_{1 \le \ell \le 3} \{g_N(q_N, \ell_{N,j}) + f_{N-1}(Q - q_N, \ell_N)\} \ldots\ldots (82)$$

Equation (82) is the algorithm used in this study to determine the optimal pairs of decisions q_i and $\ell_{i,j}$ subject to the constraints

$$\left.\begin{array}{l} q_{min} \le q_i \le q_{max} \\ \sum_{i=1}^{N} q_i \le Q \\ 1 \le \ell_{i,j} \le 3 \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (83)$$

Equation (82) and (83) form the basis of the computer program coded for this study. The program flow chart is given in Figure 13 and an outline of the various steps is presented in the succeeding paragraphs.

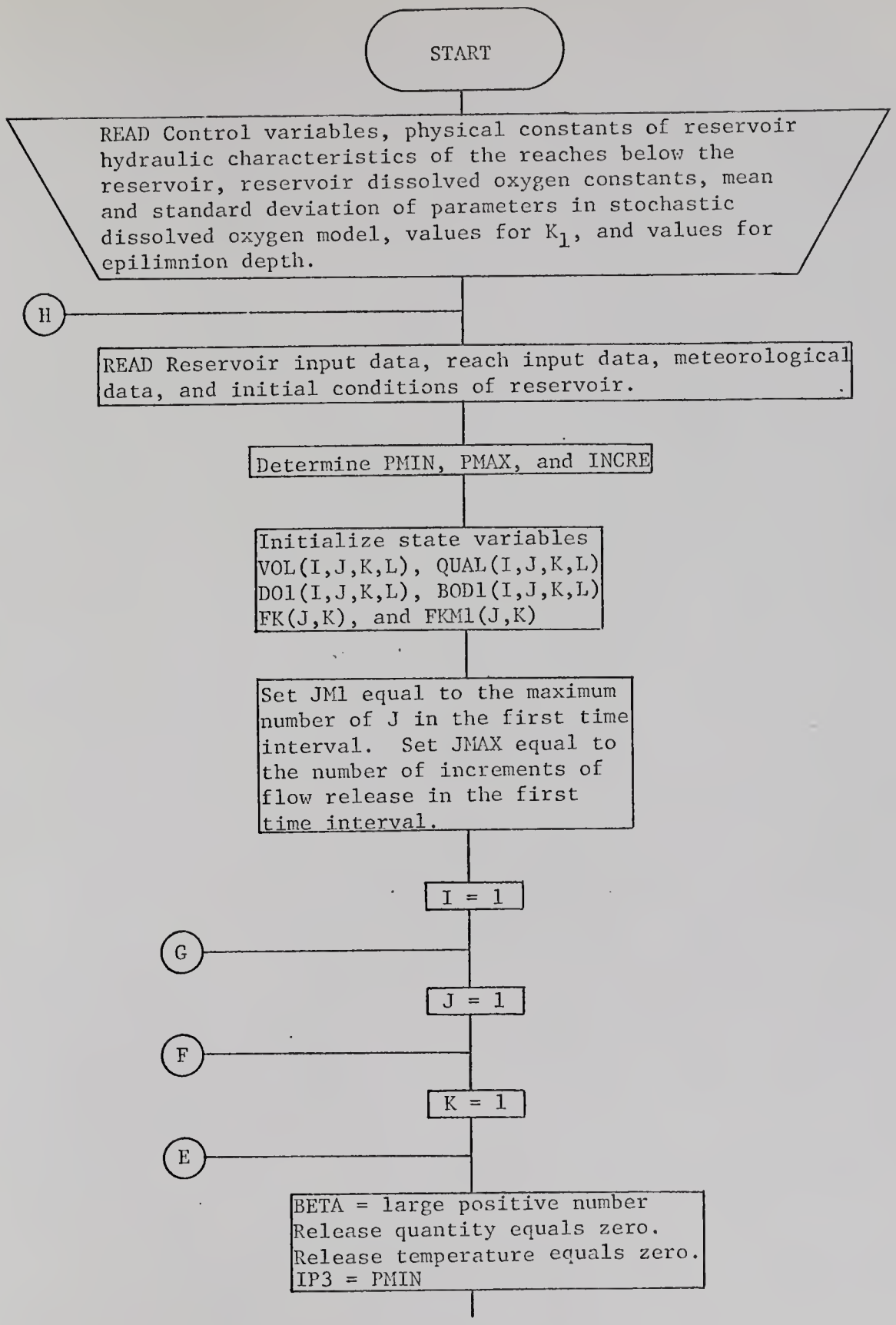

START
READ Control variables, physical constants of reservoir hydraulic characteristics of the reaches below the reservoir, reservoir dissolved oxygen constants, mean and standard deviation of parameters in stochastic dissolved oxygen model, values for K_1, and values for epilimnion depth.
H
READ Reservoir input data, reach input data, meteorological data, and initial conditions of reservoir.
Determine PMIN, PMAX, and INCRE
Initialize state variables VOL(I,J,K,L), QUAL(I,J,K,L) DO1(I,J,K,L), BOD1(I,J,K,L) FK(J,K), and FKM1(J,K)
Set JM1 equal to the maximum number of J in the first time interval. Set JMAX equal to the number of increments of flow release in the first time interval.
I = 1
G
J = 1
F
K = 1
E
BETA = large positive number
Release quantity equals zero.
Release temperature equals zero.
IP3 = PMIN

FIGURE 13. DYNAMIC PROGRAMMING FLOW CHART

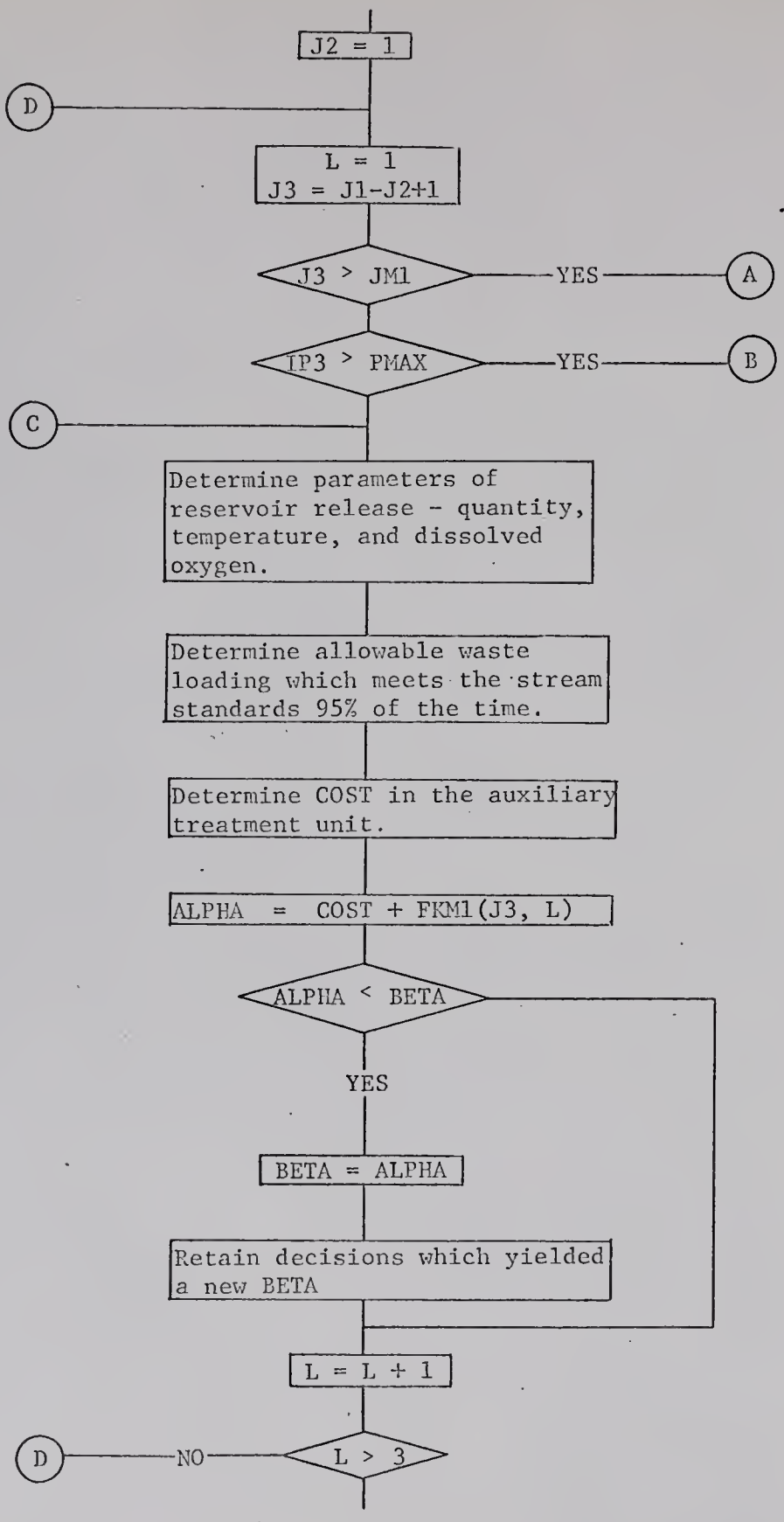


FIGURE 13. CONTINUED

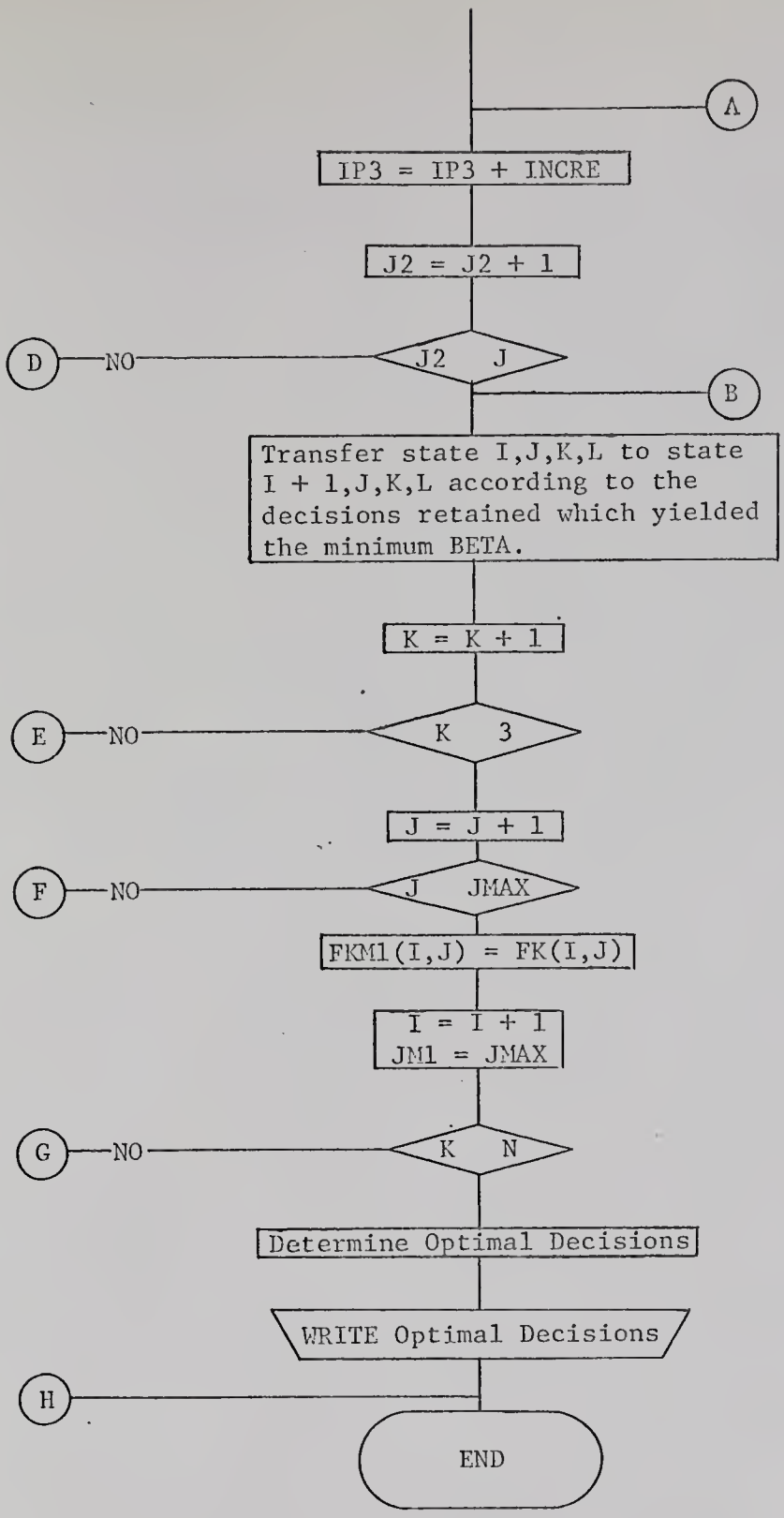


FIGURE 13. CONTINUED

Step 1. The first part of the procedure consists of reading the control variables, the physical constants of the reservoir, the hydraulic characteristics of the downstream reaches, the reservoir dissolved oxygen constants, the parameters used in the stochastic dissolved oxygen model, the values for the deoxygenation coefficient, and the values for the epilimnion depth variation with time. The control variables determine the various options which can be employed with the dynamic programming algorithm. These options are explained later in this chapter. The physical constants of the reservoir are the minimum and maximum allowable design storage for the reservoir. The hydraulic characteristics of the reaches are the constants in Equations (67) and (68) which determine the mean velocity and mean depth as a function of stream discharge rates. The reservoir dissolved oxygen constants are the values which constitute the matrix of Figure 9. The mean and standard deviation of r and s in Equations (63)-(66) as well as the constants for determining the standard deviations of K_1 and K_2 are placed in a READ statement. The values for the epilimnion depth were discussed in Chapter II. The variables mentioned above are assumed to be invariant from year to year and are, therefore, the first variables read into the computer.

Step 2. This step also involves the reading of variables, but it is limited to those that change annually. These data include the monthly flows into the reservoir, the monthly flows from tributaries downstream, and the average monthly meteorological data. The initial conditions of the reservoir, storage and isothermal temperature are included as annual data requirements.

Step 3. In this step, the total available water resource for the periods under consideration is determined by adding the reservoir inflows for each period to the initial storage. It is this total which is optimally allocated among the various intervals. The minimum flow, PMIN, to be allocated during each period and maximum flow, PMAX, are determined from the total available resource. These two flows may be governed by consideration of flood control requirements, navigation requirements, or fish and wildlife requirements. The number of discrete values of release quantities determines the incremental release rates, INCRE. For example, if PMIN were 4000 cfs and PMAX were 12,000 cfs and five increments of discharge were to be considered, then INCRE would be 2000 cfs. In this case, the five discharge rates would be 4000, 6000, 8000, 10,000, and 12,000 cfs. Computer time increases with the number of discharge rates considered; therefore, one must be careful to weigh the cost against the information obtained by considering several discharge rates.

Step 4. This step consists of initializing the state variables for the first time interval and setting the first cost matrix, FMK1, equal to zero. The major state variables are the storage in the various layers, VOL, the temperature of the layers, QUAL, and the dissolved oxygen in the layers, DO1. To illustrate the meaning of the subscripts of the state variables, VOL(I,J,K,L) is the storage in layer L resulting from the release rate J from layer K in period I-1. If in period two, the optimum decisions for a quantity of resource, Q, were to release 6000 cfs (e.g., corresponding to J equals two) from layer 3 (corresponding to K equals three), then

VOL(3,2,3,L) would be the volumes in layers L equal 1, 2, and 3 resulting from the decisions J equal 2 and K equal 3 in period two.

Step 5. Since the subscript J increases from period to period as more resource is allocated, it is important to define the limit, JM1, on J for any one time period. In the first time interval, the limit is the number of discharge rates to be considered. It also is important that the maximum limit, JMAX, for J be established. For the first interval, JMAX is the number of discharge rates considered. After the first interval, JM1 is always JMAX of the previous interval. In the example of dynamic programming given earlier, JM1 and JMAX were four for the first interval. In the second interval, JM1 was four and JMAX was seven; and in the third interval, JM1 was seven and JMAX was seven.

Step 6. The index I for the number of periods is initialized to one.

Step 7. The index J for the discharge is initialized to one. J equals one corresponds to the minimum discharge rate for the current period. When J is two, the discharge rate is PMIN + INCRE. For any J, the discharge rate is PMIN + J(INCRE).

Step 8. The layer index K is initialized to one corresponding to the epilimnion.

Step 9. BETA represents the minimum cost for decisions J and K. For each new K, BETA is initialized to a large positive value so that the first decision will force a new BETA. Also, the quantity

released, GAMMA1, and temperature of the release, GAMMA2, are initialized to zero. The discharge rate, IP3, is initialized to PMIN.

Step 10. An artificial variable, J2, is initialized to one. The limit on J2 is J.

Step 11. The layer, L, from which the release could have been made in the previous period is set to one. Since the release rate, IP3, could have been made from any of the three layers in the previous period, each of these possibilities must be considered. The value J3 is determined as J-J2+1. The values of J3 and L correspond respectively to $Q-q_N$ and $\ell_{N,j}$ of Equation (82).

Step 12. If J3 is larger than JM1, the index on J for the previous interval is exceeded and the computer skips to Step 19. Otherwise, the computer goes to the next step.

Step 13. If IP3 exceeds JMAX, optimization for decisions J and K is completed and the computer skips to Step 20.

Step 14. In this step, the quality of the water to be released is determined. These parameters are dissolved oxygen, temperature, GAMMA2, biochemical oxygen demand, and release quantity GAMMA1.

Step 15. The allowable waste loading is determined which maintains the dissolved oxygen above the minimum allowable dissolved oxygen standard 95% of the time.

Step 16. The cost of treating the waste to meet the allow-

able loading is calculated. This is the cost of treatment in the auxiliary unit.

Step 17. The total cost of treatment for allocating the resource Q is determined by Equation (82). This cost is represented by ALPHA.

Step 18. If ALPHA is less than BETA, the computer goes to Step 19. Otherwise, it skips to Step 20.

Step 19. BETA is set equal to ALPHA and the decisions on GAMMA1 and GAMMA2 which yielded the new minimum are retained.

Step 20. In this step, the next layer is considered and the computer goes back to Step 13 if L is not greater than three. When L exceeds three, the computer goes to the next step.

Step 21. A new value for IP3 is calculated and J2 is incremented by one. If the new value of J2 is not greater than J, the computer goes back to Step 11.

Step 22. When J2 exceeds J, the computer transforms the reservoir according to the decisions which yielded the minimum BETA and stores the values of GAMMA1 and GAMMA2.

Step 23. The value of K is incremented by one and the computer goes back to Step 9 with instructions to release from the next layer.

Step 24. When K is greater than three, a new value for J is calculated. This increases the available resource allocated to the

current time interval. The computer then proceeds to Step 8. The decisions concerning from which layers the resource should be removed are determined for the new J.

Step 25. When J exceeds JMAX, the optimization for period I is complete. The FKM1(I,J) matrix is transferred to the FK(I,J) matrix for use in the next interval. The FK(I,J) matrix corresponds to the left side of Equation (82) and the matrix FKM1(I,J) corresponds to the function $f_{N-1}(q,\ell)$. The value for I is incremented by one and the procedure for the next period is repeated by going back to Step 7.

Step 26. After the optimal decisions have been made for the last interval, the minimum total cost in the matrix FKM1(I.J) corresponds to the optimum set of decisions from period one to the last period. By beginning at this point and tracing through the decisions stored in Step 20, these optimal decisions are determined.

Step 27. The optimal decisions are printed in the output and the computer proceeds to the next year by going to Step 2.

The computer listing of the dynamic programming algorithm is given in Appendix B along with the subroutines and definitions of terms. For a more detailed discussion of the development of dynamic programming, the reader is referred to Bellman and Dreyfus.[63]

Alternatives

The use of the control variables mentioned in Step 1 provides the methodology with a capability of evaluating several reservoir alternatives. Since the programming is coded for a two-dimensional

algorithm, the first alternative is that of considering a multi-level outlet structure for release in any layer. To do this, the number of layers, NL, is set to three. If reaeration of the reservoir outlet is to be considered, the variable IAERAT is made positive and the level of reaeration to be used is the variable DORELS. Thus, the costs of waste treatment for various dissolved oxygen levels maintained in the reservoir release can be compared.

The alternative of one outlet in the hypolimnion can be evaluated by forcing the values for K and L in Steps 8 and 11 to be three. This is done by making the variable IK8 positive. The maintaining of various dissolved oxygen levels can be considered by use of the variables IAERAT and DORELS. In this manner, the costs of waste treatment with a multi-level outlet can be compared with those for a single outlet.

Another reservoir alternative is to mix the reservoir to an isothermal condition. This is done by making NL equal one. Thus, the mixed reservoir can be compared to the stratified reservoir by evaluating the costs of waste treatment for the various alternatives.

Each of the three reservoir alternatives can be evaluated using the same dynamic programming formulation. However, in the cases of one outlet and a mixed reservoir, a fundamental change in the algorithm takes place. When only one outlet is used, the decision of considering release from the other layers is foregone, and the two-dimensional algorithm is transformed to a one-dimensional algorithm for optimizing the release quantity. The reservoir remains stratified, but only release from the hypolimnion is considered. The same trans-

formation of the algorithm occurs when the reservoir is mixed because releases from any level in the reservoir have the same temperature.

The methodology proposed in this dissertation provides a means for evaluating several reservoir water quality alternatives by comparing the waste treatment cost of the auxiliary unit for each alternative. The fundamental question for the basis of this methodology is: If the reservoir inputs and meteorological factors are known, how should the available resource be allocated over the summer months to minimize the cost of meeting the stream standards? By maintaining optimal allocation of the resource for the various alternatives, the least-cost alternative can be found. Then the question can be asked: How should the reservoir be operated using the least-cost alternative in the case of uncertainty of inflow and meteorological data? In the next section, a method for determining optimal release rules will be discussed.

Operating Rules

As pointed out earlier, Young[61] used Monte Carlo Dynamic Programming to obtain release rules for flood control reservoirs. The rules were obtained by determining optimal releases for various traces of simulated flows. When this was done, a set of observations of optimal releases for various conditions of inflow and storage were recorded. The optimal release was assumed to be a linear function of the reservoir storage and inflow in the preceding interval. A regression analysis on the generated data and storage yielded the release rule. Young found that forecasting the inflow for the next period gave better multiple correlation for the release in the current period.

The Monte Carlo Dynamic Programming technique appears to be applicable to determining release rules, and it is suggested that this technique be employed to answer the question of reservoir operation under uncertainty. After determining the least-cost alternative, the reservoir operation could then be simulated for several years of synthetic flows and meteorological data. The optimal sets of releases could be recorded along with the reservoir storage and previous inflows. Forecasts could be made for each inflow and monthly operating policies established by performing a regression analysis of releases as a function of storage, inflow, and forecasts.

In this chapter, the methodology for evaluation of alternatives was presented. The technique required the use of a two-dimensional dynamic programming algorithm for obtaining the optimal decisions for release quantity and layer. The methodology is demonstrated in Chapter VII on a test case. Young's method was proposed for obtaining the release rules for each time period. In the next chapter, the data requirements of the methodology are outlined.

VI. DATA REQUIREMENTS

The various models described in Chapters II and III for predicting reservoir and river responses require a certain amount of data. The data requirements can be grouped into four categories: reservoir, river, waste, and meteorological. Each of the four groups will be discussed with respect to the type of data needed and the purpose of the data.

For existing reservoirs, in situ observations of dissolved oxygen and temperature profiles over several years are needed. The measurements provide the data necessary for determining the variation of the epilimnion depth with time and the reservoir dissolved oxygen constants. The volumes of inflowing water and their temperatures provide input to the reservoir temperature model. Also, the biochemical oxygen demand of the reservoir releases should be known for use in the stream dissolved oxygen model. The area-capacity curves for the reservoir are needed for calculation of evaporation and volumes of the different layers in the reservoir.

To use the reservoir model for evaluating alternatives for proposed reservoirs, one must assume the dissolved oxygen constants discussed at the end of Chapter II. The epilimnion depth might be assumed constant or an analysis of reservoir stratification using the Huber-Harleman Model[28] might be performed to determine variation of the epilimnion depth.

The river data include the downstream tributary flows and temperatures, the distance between reaches, and the constants of Equations (66) and (67) for determining the mean depth and mean velocity in each reach. The biochemical oxygen demand and dissolved oxygen of the tributary flows are quality data used in the dissolved oxygen model. The composite parameters, r and s, and their variance also are needed in the dissolved oxygen model. The stream standards for the various reaches are an integral part of the determination of the allowable waste load. The minimum reservoir release to maintain navigation, fish and wildlife, or any other purpose should be known to establish one of the constraints in the reservoir release policy.

The volume and concentration of the municipality's waste discharge must be known to calculate the waste treated in the auxiliary unit. The auxiliary waste treatment process and its operating cost establish the cost of this unit. It is most desirable to have accurate auxiliary waste treatment costs since this cost is minimized and since the methodology is established for comparing costs of alternatives.

The meteorological data are inputs needed to calculate the equilibrium temperature used in the reservoir and stream temperature models. These data include air temperature, air vapor pressure, cloud cover, wind velocity, and short-wave solar radiation. If the downstream climate is much different from the reservoir climate, then two sets of meteorological data must be used to adequately determine the equilibrium temperature in the two models.

Since the stream models are based on steady, uniform flow, the data should be averaged over time intervals for which the flows can be considered constant. There is a practical limit in the methodology to the number of periods which may be considered. The methodology is applied to the seven-month period from April through October. The largest time interval should be no longer than a month so that adequate representation of reservoir dynamics is maintained. The time of flow through a basin determines the minimum time interval, since predicting water quality for periods less than this would violate the steady-state requirements mentioned in Chapter III. If the travel time approaches one week, then probably bi-weekly intervals would be sufficient to represent the system. In allocating the water to the various time periods, it is not necessary that the interval for each period be identical. Therefore, a monthly interval might be chosen for the first three or four periods and a bi-weekly interval selected during the critical months. It should be mentioned that increasing the number of periods increases computer time. By weighing the cost of computer time against the additional information obtained from consideration of several time periods, a decision can be made as to how many periods should be included in the model. This provision of varying time intervals gives the users of the methodology some flexibility in the selection of the number of periods.

In the final analysis, the selection of time intervals is limited more by the data sampling interval than the time of travel in the basin. If the average of the data for the intervals is based on one or two data observations, then the average might not be an adequate representation and a larger time interval should be selected. The

availability and sources of the data needed for this methodology are discussed in the next chapter as the test case is developed.

VII. TEST CASE

The purpose of this chapter is to illustrate the use of the methodology outlined in Chapter V. Such a chapter might consist of defining a hypothetical river basin with appropriate assumptions for inflows and data; however, an actual river basin is selected in order to more accurately demonstrate the steps necessary for applying the methodology. Since inadequate data acquisition is always a problem, several assumptions are made to fill the holes in the input to the various models. The description of the test case and list of necessary assumptions are presented in the following sections.

Description of River Basin

The basin selected for illustration of the methodology was the Savannah River from the Clark Hill Reservoir, at river mile 237.3, to river mile 37.5 above Savannah, Georgia. The Clark Hill Reservoir has a normal surface area of 70,000 acres and a volume of 2,600,000 acre-ft. It was designed and constructed by the U. S. Army Corps of Engineers for the purposes of flood control, generation of hydroelectric power, and for maintaining sufficient flows for navigation. Since construction of the dam in 1951, the Corps has maintained several data collection programs both at the reservoir site

and downstream. The data include dissolved oxygen and temperature profiles in the reservoir and dissolved oxygen and temperatures at various stations downstream from the reservoir. In addition, stream flow measurements have been made by the U. S. Geological Survey for several years at gaging points throughout the basin.

The city of Augusta, Georgia is located about thirty-five miles downstream from the reservoir. In 1960, the city population was about 100,000 people with a suburban population of an additional 100,000 people.[64] Several small communities and an Army post add to the population. Major industries such as pulp and paper, textile, and food processing are located at Augusta.

The average annual flow of the Savannah River at Clark Hill Reservoir is 8,400 cfs. There are two major tributaries to the Savannah River below Clark Hill Reservoir: Stephens Creek and Brier Creek. The confluence of Stephens Creek and the Savannah River is 12.8 river miles below the dam. The average annual flow of Stephens Creek is 365 cfs. Brier Creek flows into the river 132.3 miles below the dam and has an average annual flow of 684 cfs.

In accordance with the Water Quality Act of 1965, the state of Georgia established stream water quality standards. The dissolved oxygen standard for the Savannah River below Clark Hill is 4.0 mg/l.[65] The river-reservoir basin described above meets the requirements described in Chapter V for which the methodology was developed. An outline of the basin is given in Figure 14.

Reservoir Data

The reservoir data collected by the Corps of Engineers

FIGURE 14. SAVANNAH RIVER BASIN

enabled the determination of the reservoir dissolved oxygen constants and the variation of epilimnion depth. The dissolved oxygen profiles for the summer months of 1967 were used to determine the reservoir dissolved oxygen constants. Three of these profiles are shown in Figure 8a. The temperature profiles taken in 1956, 1957, 1964 and 1967 were used for the determination of the variation of epilimnion depth with time. The reservoir dissolved oxygen constants and epilimnion variation are given in Table 7.

TABLE 7

RESERVOIR DISSOLVED OXYGEN CONSTANTS
AND EPILIMNION DEPTH VARIATION

Time	Dissolved Oxygen Constants in Per Cent Saturation			Epilimnion Depth in Feet
	Epilimnion	Metalimnion	Hypolimnion	
April	95	95	95	5
May	95	85	60	15
June	95	70	40	20
July	95	60	20	25
August	95	40	20	30
September	95	40	20	40
October	95	50	20	60

The inflow volumes were recorded in the Summary of Clark Hill Operations[66] as well as estimated evaporation and release volumes. The Corps recorded downstream water temperatures in 1956, 1957, and 1958. These data indicated that release temperatures reached approximately a constant value after some 172.3 miles. A continuous

temperature recorder has been in use at this location since 1956. The monitor is near Clyo, Georgia. There have been very few observations of inflowing temperature at Clark Reservoir. Since the inflow would probably be at some equilibrium value, it was assumed that average monthly temperature recordings at Clyo would be approximately equal to the inflows at the reservoir.

There have been a few BOD and DO measurements reported on the Savannah River and its tributaries.[67] The data indicate BOD values of less than 2.0 mg/l and DO levels of at least 85% saturation. Since no continuous monitoring is made of BOD and DO within the basin, all reservoir releases and tributary flows were assumed to have a BOD of 2.0 mg/l and all DO levels of the tributary flows were assumed to be at 85% saturation.

Stream Data

The Savannah River was divided into ten reaches. The reaches were established at changes in the river's slope, at confluences with tributaries, at locations with cross-section data, at the point of waste discharge, and at approximately twenty-five-mile intervals when none of the aforementioned reasons occurred.

Two cross-section soundings were available from the U. S. Geological Survey at river miles 129.2 and 65.0. Using techniques of open channel flow, the river cross-section data were assumed to be approximated by a fourth order polynomial. By routing various flows through the basin and using an equation similar to Manning's Equation, having a corresponding Manning roughness of 0.056, the mean depth and mean velocity, as a function of discharge, were found for each of

TABLE 8

RIVER HYDRAULIC DATA

REACH	α_v*	α_d*	β_{dv}*	β_d*	DISTANCE
1	.1156	.08547	.3465	.5214	12.8
2	.1838	.05848	.3469	.5217	4.5
3	.09747	.09709	.34705	.5216	8.0
4	.09747	.09709	.34705	.5216	10.0
5	.07518	.1192	.3476	.5218	24.0
6	.07518	.1192	.3476	.5218	24.0
7	.07518	.1192	.3476	.5218	24.0
8	.07518	.1192	.3476	.5218	25.0
9	.07518	.1192	.3476	.5218	40.0
10	.07518	.1192	.3476	.5218	27.5

*Constants in Equations (66) and (67)

the reaches. The values for the constants of Equations (66) and (67) and the reach distances are given in Table 8. The constants are identical for reaches having the same slope. Examination of Table 8 shows that there are four distinct changes in bed slope. The Manning roughness value of 0.056 corresponds to sluggish rivers. Aerial photographs of the Savannah River indicate the river follows a tortuous path through the basin. This is confirmed by observing that the overland distance from Augusta to Savannah is one hundred eight miles whereas the river distance is one hundred ninety-seven miles.

Very few temperature observations on Stephens and Brier Creek have been made. To be consistent, the temperatures of these tributaries were assumed to be approximated by the values at Clyo. The inflowing DO and BOD values were mentioned in the previous section.

The average annual runoff at Clyo is 11,840 cfs. When the average annual runoff at Clark Hill, Stephens Creek, and Brier Creek are added, there remain some 2,391 cfs. This additional runoff was apportioned to the various reaches according to the incremental drainage area for each reach. The values for the ungaged runoff are shown in the table below.

TABLE 9

DISTRIBUTION OF UNGAGED RUNOFF

Reach	Drainage Area sq. mi.	River Mile	Runoff cfs
1	6150	237.3	Reservoir Discharge
2	545	224.5	Stephens Creek
3		220.0	377
4		212.0	Waste Discharge
5	7508	202.0	398
6		178.0	263
7		154.0	363
8	8650	130.0	363
9	645	105.0	Brier Creek
10	9850	65.0	529

Meteorological Data

Average monthly meteorolgical data were found in the National Summary of Climatolgical Data.[68] The monthly averages of air temperature, relative humidity, cloud cover, and wind speed at Augusta, Georgia were assumed valid for Clark Hill Reservoir.

The short-wave radiation values were obtained by linear interpolation of the recordings at Charleston, South Carolina, and Atlanta, Georgia. The air-vapor pressure was obtained from the relative humidity and air temperature readings using a psychrometric chart.

Model Verification

It was felt that some attempt should be made to see if the models outlined in Chapters III and IV yielded reasonable predictions of reservoir and river temperatures. Therefore, several tests were made using existing data to determine the agreement between observed and predicted temperatures.

Several years of monthly reservoir temperature profiles were taken after the reservoir was built. The equilibrium temperature was determined for each of the months using the meteorological data described above. The observed temperatures and calculated equilibrium temperatures are shown in Figure 15 for data collected from 1953 through 1954. The surface water temperature is observed to lag the equilibrium temperature as pointed out in Chapter III. The results indicate that the equilibrium temperature can be used for estimating the surface water temperature. More confidence can be placed in the estimate during the summer months, which is the period of primary interest. It appears that the assumptions made in the use of the meteorological data will provide a reliable estimate of the surface water temperature.

The Corps of Engineers conducted temperature surveys downstream from Clark Hill Reservoir from June 1955 to October 1958. The data include average monthly temperatures in the tailwater. The

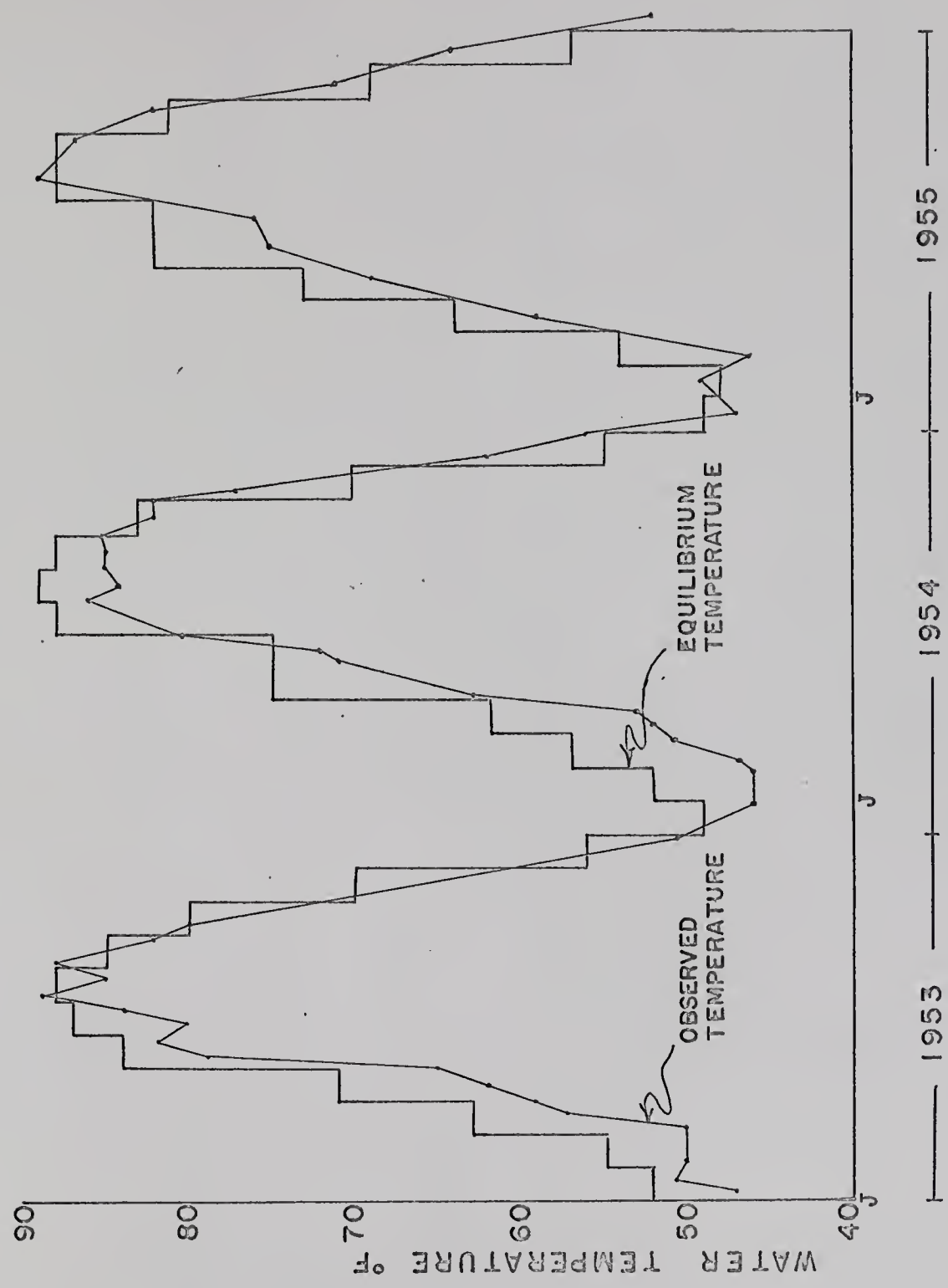


FIGURE 15. RESERVOIR SURFACE TEMPERATURES

values for the heating period in 1958 were used to test the reservoir release temperature model. The reservoir withdrawal volumes were taken from the Summary of Clark Hill Operations. The results of the test are given in Table 10.

TABLE 10

RESERVOIR RELEASE TEMPERATURES

Month	Inflow Volume cfs	Outflow Volume cfs	Inflow Temp* °F	Release Temperatures Observed °F	Predicted °F
April	17410	17314	61	53	56
May	9757	10237	70	58	56
June	5156	6023	77	62	56
July	7598	6840	78	65	60
August	4328	6602	80	69	69
September	2598	5157	78	73	71
October	2647	5110	70	71	72

*Temperatures at Clyo

The agreement between predicted and observed temperatures is fairly good during the early spring with good results in the latter part of the year. The discrepancies in the early part of the year probably are due to the assumption that the hypolimnion temperature remains constant; it probably warms slightly during the heating period. However, the overall agreement is such that the reservoir model provides an adequate representation of the reservoir dynamics.

The downstream temperature studies made by the Corps provided an opportunity for checking the stream temperature model. A summer month was selected for the test since this is usually the critical time. The downstream profiles were predicted for various release temperatures in increments of five degrees. Figure 17 shows the comparison of the observed data with release temperatures at 65 and 70°F. Each profile has approximately the same shape as the observed profile. The predicted profiles are approaching an equilibrium temperature somewhat higher than that of the observed profile. The discrepancy is probably because the equilibrium temperature at the reservoir is not the same as that two hundred miles downstream. It is felt, however, that the agreement is certainly adequate for the purpose of this study.

Waste Discharge Model

In 1960, the municipal and industrial waste discharge which could be considered to enter the Savannah River near Augusta, Georgia had a population equivalent of 335,157.[64] Assuming 0.18 lbs. of 5-day BOD per person per day yields a daily waste load at Augusta of 60,328 lbs. of 5-day BOD. It was felt that such a load would hardly be enough to tax the oxygen resources of the Savannah River. Therefore, several computer runs were made using the stream temperature and dissolved oxygen models (with various release temperatures and flows) for the purpose of ascertaining the allowable waste loading of the Savannah River. In doing this, a minimum base flow was calculated for the reaches below Clark Hill. The assimilative capacity, therefore, would be primarily from the reservoir releases

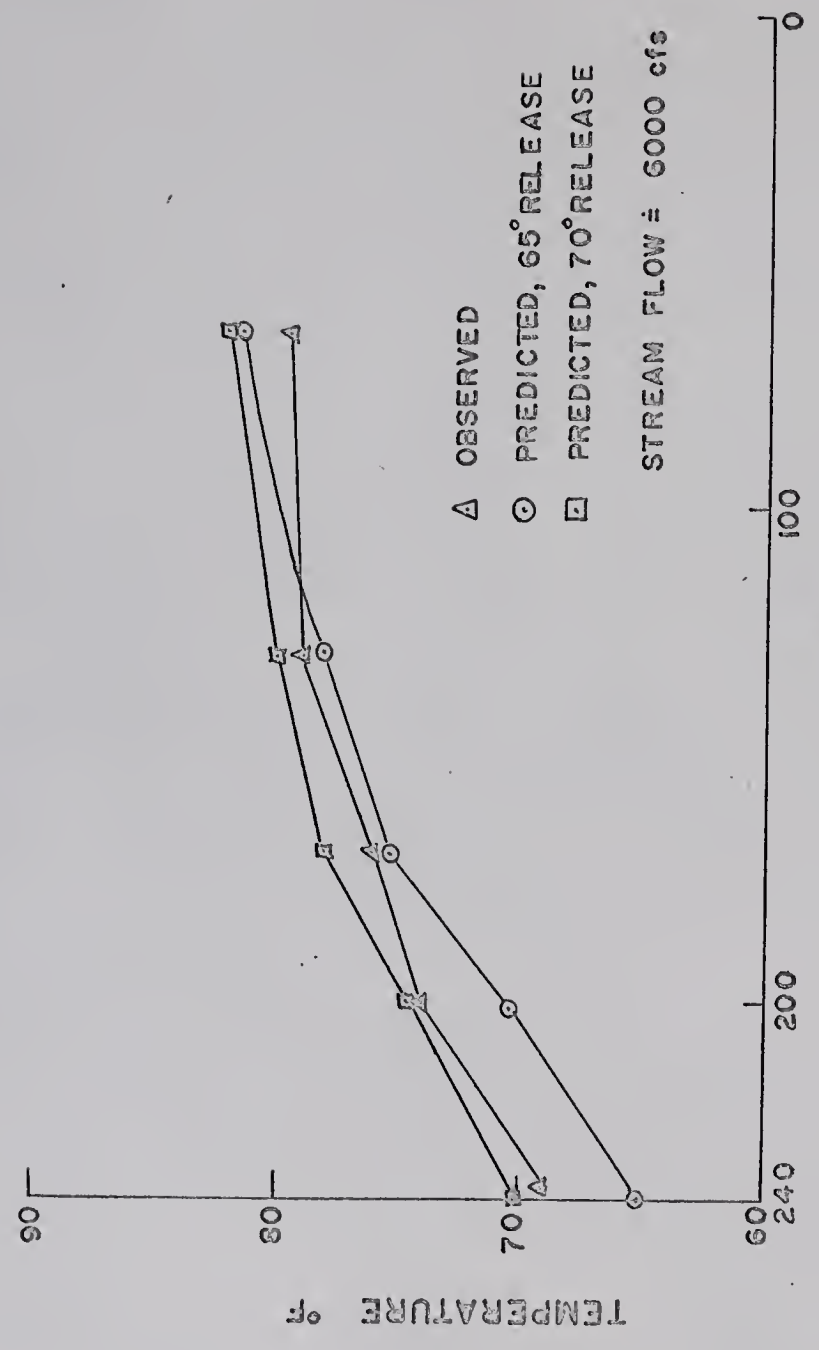


FIGURE 16. DOWNSTREAM TEMPERATURE PROFILES
for AUGUST 1957

and not from the tributary flows. The base flows were determined from stream flow measurements in the summer of 1958. That year was chosen because very little rainfall occurred and flows were almost constant. The base flows are listed in Table 11.

TABLE 11

BASE FLOW

Reach	1	2	3	4	5	6	7	8	9	10
Base Flow, cfs	Reservoir	5	360	Waste	380	55	55	55	205	67

The allowable waste load was found as that load which would satisfy the dissolved oxygen standard 95% of the time. To simulate actual conditions, 1958 meteorological data were used to determine the equilibrium temperature needed in the stream temperature model. Figures 17 and 18 illustrate part of the results. The curves shown in Figure 17 were for releases with dissolved oxygen levels maintained at 6.0 mg/l and an equilibrium temperature, T_E, of 86°F. The shapes of the curves in Figure 17 exhibit the features of the hypothetical curves in Figure 11. A tradeoff between temperature and flows is clearly demonstrated for the Savannah River. The slopes of the curves for constant release flows are greater than those for constant temperatures, indicating that temperature has a more important effect on waste assimilation than stream flows.

The curves in Figure 18 illustrate the effect of equilibrium temperature and dissolved oxygen on allowable waste loading. The allowable waste loading is observed to decrease with increasing

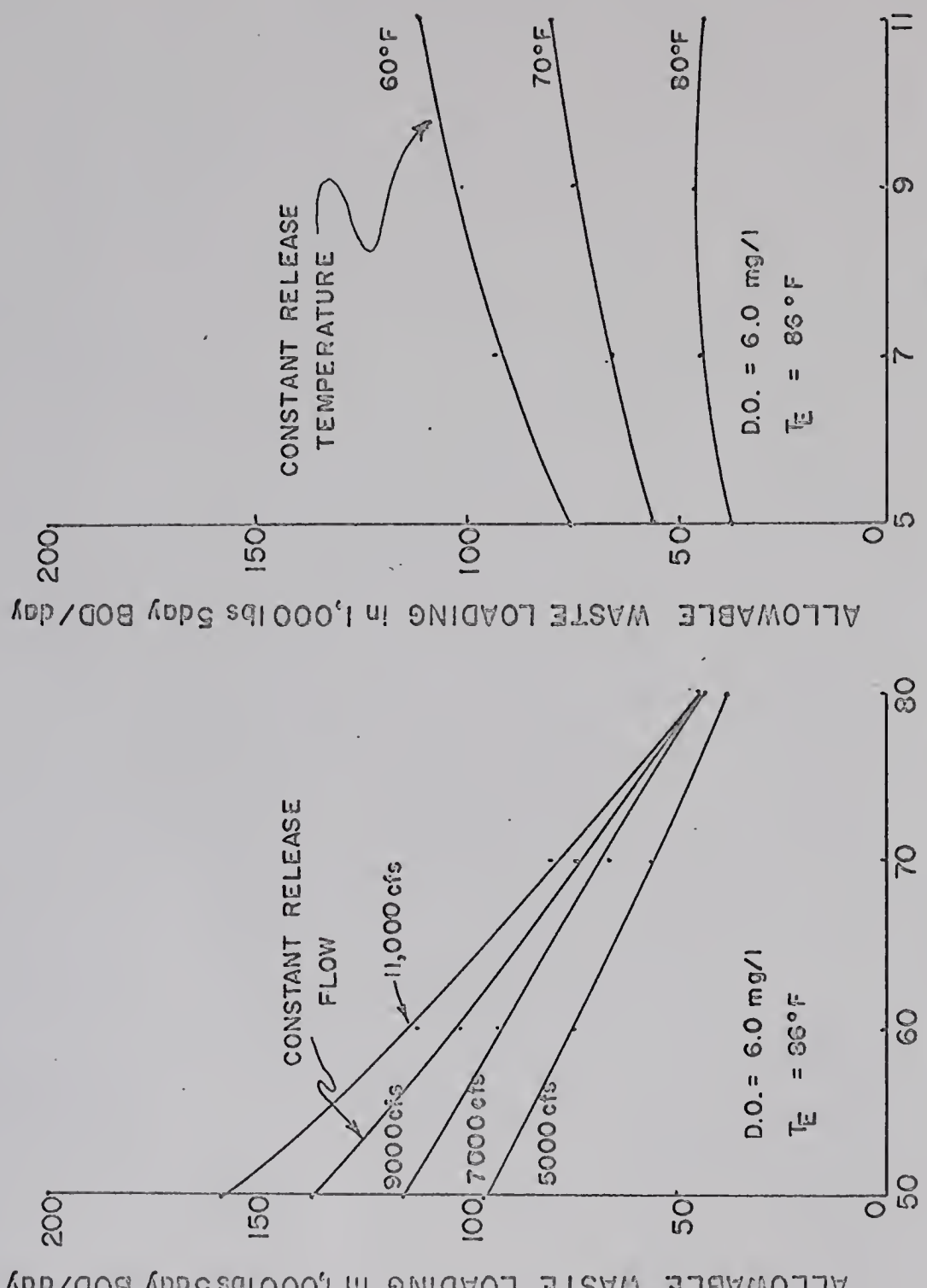
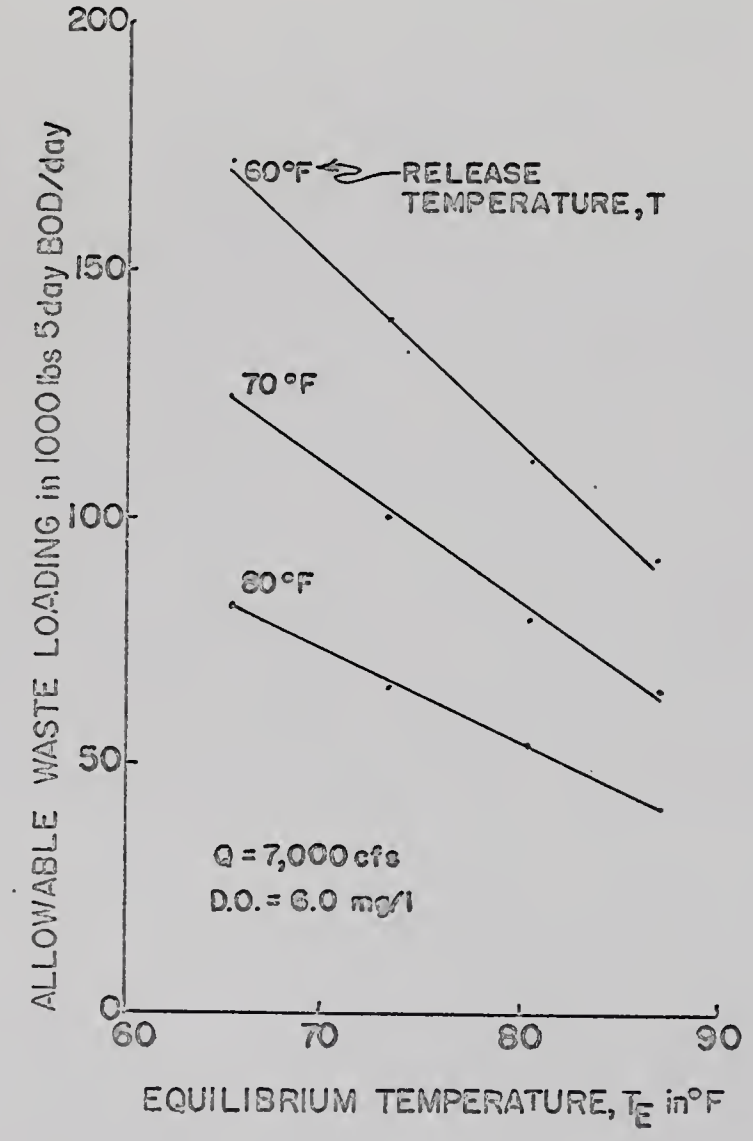


FIGURE 17. WASTE ASSIMILATIVE CAPACITY vs. TEMPERATURE and FLOW



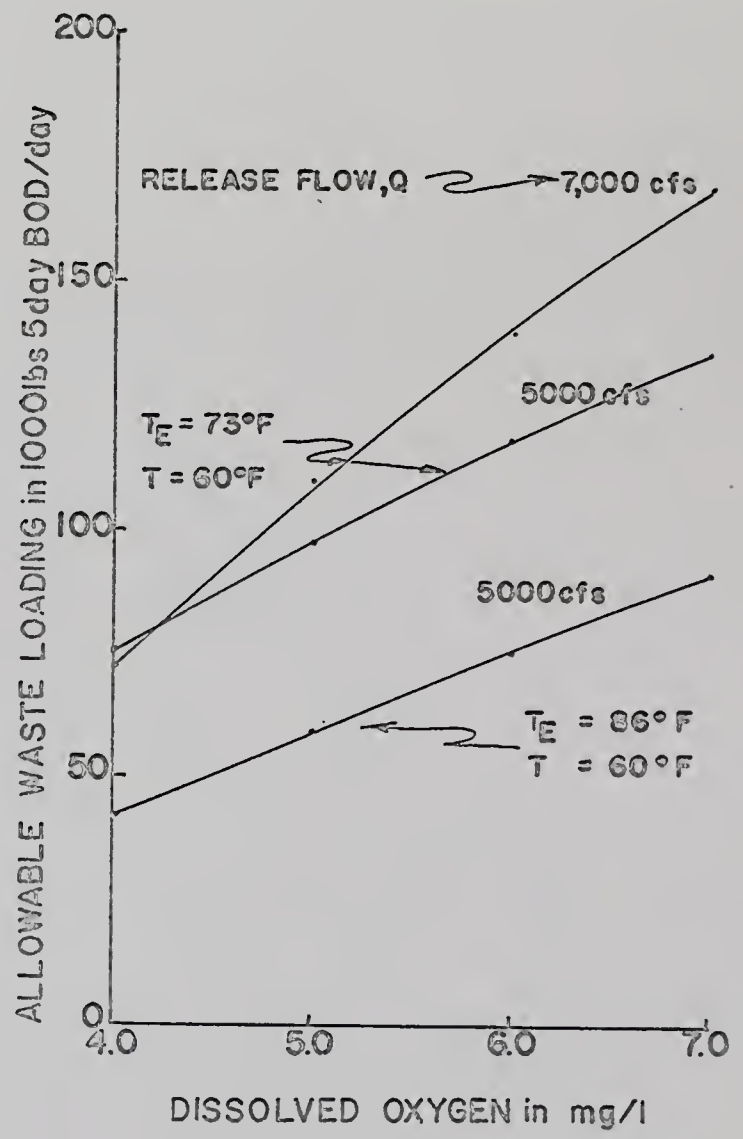


FIGURE 18. WASTE ASSIMILATIVE CAPACITY vs. EQUILIBRIUM TEMPERATURE and DISSOLVED OXYGEN

equilibrium temperature. This effect corresponds to decreasing assimilative capacity during the summer months. The temperature tradeoff is quite evident for changing equilibrium temperatures. Increasing the dissolved oxygen levels in the reservoir release increases the stream's assimilative capacity, as shown by the curves on the right in Figure 18. Certainly this would be expected when reservoir water quality has such a marked effect on waste assimilative capacity.

The results of the computer runs for determining the waste assimilative capacity of the Savannah River indicated marked variation of waste loading with equilibrium temperature, dissolved oxygen, and release temperature, but only slight variation with stream flow. A linear regression was performed for the waste assimilative capacity as a function of equilibrium temperature, dissolved oxygen, release temperature, and stream flow. It was found that the linear model accounted for 91% of the variance of the waste assimilative capacity. The results of the regression are given in Table 12. Analysis of these results would lead one to conclude that the best method of improving the waste assimilative capacity of the Savannah River would be through temperature and dissolved oxygen control. It would seem that since release flows contribute so little to the total variance of the waste assimilative capacity in the Savannah River a larger reservoir for flow augmentation might not be the best method of improving water quality.

Auxiliary Treatment Unit

Since the assimilative capacity of the Savannah River almost always exceeded the 1960 waste loading, a larger loading was assumed

TABLE 12

LINEAR REGRESSION RESULTS

Variable	Proportion of Variance per cent	Total Variance per cent
Equilibrium Temperature	31.2	31.1
Dissolved Oxygen	25.6	56.7
Release Temperature	30.1	86.8
Release Flow	4.2	91.0

so that the methodology of Chapter V might be applied. The population equivalent was increased to 750,000 yielding a 5-day BOD of 135,000 lbs. The deoxygenation coefficient, k_1, was assumed to be 0.3 day^{-1}, according to recent investigations.[69] Thus, the ultimate load is 157,000 lbs BOD/day. Primary treatment was assumed to remove 35% of the BOD, and the per capita sewage volume was assumed to be one hundred gallons, yielding a waste to be discharged into the stream of 75 MGD and 102,000 lbs of ultimate BOD. An auxiliary unit was selected to treat this waste to maintain the allowable loading which would satisfy the stream standard 95% of the time. A chemical treatment unit was selected since it could be easily brought "on line" when needed. The unit was assumed to remove 75% of the BOD from the waste volume treated. Operating costs of the unit were taken as approximately \$0.03/1000 gallons treated.[70] This cost might be compared to \$0.015/1000 gallons for a 75 MGD trickling filter plant. The capital cost of the trickling filter plant would be \$15,000,000, whereas the chemical treatment plant would be \$2,500,000.[70]

No data could be found from which estimates of the parameters in the dissolved oxygen model could be obtained or verified. The value for k_1 was selected as 0.3 day^{-1} and Equation (49) was used to estimate the reaeration constant, k_2. The means of the values for r and s in Equations (58) and (59) were assumed zero, but with variances of 0.25.

The remaining assumptions were made concerning the reservoir storage capacities and release rates. The maximum allowable storage was taken as 3,400,000 acre-ft and the minimum allowable storage as 1,600,000 acre-ft. The minimum allowable discharge was assumed to be 4,000 cfs and the maximum discharge was arbitrarily set at 16,000 cfs.

Because of the constraint of average monthly meteorological data, the time interval selected for the test case was one month. This does not forego any of the uses of the methodology, but it does have the significant attribute of reducing computer time required to make an annual run.

Discharge Rates

Since there were an infinite number of possible discharge rates, it was necessary to limit the number which could be considered. The number of discharge rates had two effects on the computer usage of programs: the storage requirements and computer time increase with the number of discharge rates. Increasing the number of rates increases the possible decisions which can be made and, therefore, the amount of computer memory necessary to store the decisions. In the case of

considering multi-level outlets, the machine time required for considering two rates was approximately 1.5 minutes, while three rates required about 3.5 minutes (the computer at the University of Florida is an IBM-360). Thus, the number of discharge rates was limited to two for multi-level outlet runs and four for single outlet and mixed reservoir runs.

The discharge rate was limited by the maximum allowable and minimum allowable rates. To consider only these two would not have been realistic. The total available resource which could be allocated was found by adding the reservoir inputs to the difference in required storage levels between April and October. There required levels were obtained from the Corps' operation guide. An average rate was then found from the available resource. In the case of two rates, PMIN was 1500 cfs less than the average and PMAX was 1500 cfs greater than the average. For four rates, PMIN was 2000 cfs less and PMAX 4000 cfs larger.

Alternatives Examined and Results

To illustrate the use of the methodology for several alternative schemes of water quality control, two years of record were chosen. It was felt that two years should be selected in order to compare variation of the costs between years. The years 1958 and 1961 were selected. The data for these two years are shown in Table 13.

The alternatives considered were a single outlet in the

TABLE 13

DATA LISTING FOR TRIAL RUNS

Year	Month	Reservoir Flow cfs	Input Temp. °F	Tributary Flows Stephens Cr. cfs	Brier Cr. cfs	Air Temp. °F	Vapor Pressure mm hg	Wind Speed mph	Cloud Cover Tenths	Short-Wave Radiation Langleys
1958	April	17410	61	1168	1617	63.6	12.4	8.1	6.1	450
	May	9757	70	219	698	71.3	15.7	5.9	7.1	537
	June	5156	77	59	385	78.6	20.8	6.7	6.3	556
	July	7598	78	358	562	81.4	23.2	5.7	7.4	535
	August	4328	80	37	214	80.9	22.5	5.5	6.1	497
	Sept.	2598	78	6	184	75.8	18.5	6.2	5.4	445
	Oct.	2647	70	4	199	61.5	11.7	5.4	5.5	342
1961	April	15389	61	- *	2304	59.0	10.0	10.1	4.8	525
	May	4316	71	-	1336	68.8	15.0	5.5	6.6	560
	June	7300	76	-	498	76.2	18.5	6.1	7.0	540
	July	7575	76	-	483	80.6	21.7	5.3	6.1	558
	August	8193	78	-	518	78.7	21.7	5.4	7.7	440
	Sept.	6073	76	-	445	75.5	19.0	5.4	5.0	450
	Oct.	2775	70	-	235	62.1	11.5	5.2	3.0	400

Storage on March 31, 1958 = 2,580,980 acre-ft; Temperature on March 31, 1958 = 56°F
Storage on March 31, 1961 = 2,538,660 acre-ft; Temperature on March 31, 1961 = 60°F

*Flows at Stephans Creek were unattainable for 1961.

hypolimnion layer, a multi-level outlet, and a mixed reservoir. Two dissolved oxygen levels were imposed on the releases from the reservoir. Since the dissolved oxygen standard was 4.0 mg/1, a release which violated this standard was not allowed. For some runs, the minimum dissolved oxygen was set at 5.0 mg/1. It was assumed that some form of aeration would provide the necessary dissolved oxygen when releases violated the standard.

Table 14 shows the costs of the various alternatives considered. It is seen that a multi-level outlet with aeration to 5.0 mg/1 yielded

TABLE 14

MINIMUM TOTAL COSTS FOR VARIOUS ALTERNATIVES

Year	Run	Alternative	Minimum DO Level in Release mg/1	Aeration Level mg/1	Discharge Rates Considered	Cost
1958	A	Multi-level Outlet	5.0	5.0	2	$ 68,000
1958	B	Multi-level Outlet	4.0	4.0	2	128,000
1958	C	Multi-level Outlet	--	--	2	134,000
1958	D	Single Outlet	--	--	4	212,000
1958	E	Single Outlet	4.0	4.0	4	205,000
1958	F	Single Outlet	5.0	5.0	4	121,000
1958	G	Mixed Reservoir	--	--	4	130,000
1961	H	Multi-level Outlet	5.0	5.0	2	74,000
1961	I	Single Outlet	5.0	5.0	4	126,000
1961	J	Mixed Reservoir	--	--	4	89,000

the lowest cost. Marked improvement in assimilative capacity resulted from higher dissolved oxygen levels in the release water. The multi-level outlet permits the saving of the hypolimnion water for release during the summer months. Table 15 shows the optimum decisions which yielded the minimum costs. The decisions can be compared to those

TABLE 15

OPTIMUM DECISIONS FOR RUN A

Period	Release Volume Acre-ft	Layer Released From	Temperature °F
April	368,977	1	69
May	450,543	3	56
June	549,918	2	65
July	549,918	3	56
August	368,977	3	56
September	549,918	3	56
October	368,977	2	60

of Run C shown in Table 16. In Run C the releases were made from the epilimnion after the oxygen level in the hypolimnion decreased to less than 4.0 mg/1. By providing aeration to 5.0 mg/1, releases from the hypolimnion could be made during the summer months, which resulted in significant savings. The cost of providing this aeration could be balanced against the savings to see which would be the cheaper alternative.

TABLE 16

OPTIMUM DECISIONS FOR RUN C

Period	Release Volume Acre-ft	Layer Released From	Temperature °F
April	368,977	1	69
May	450,543	3	56
June	549,918	2	65
July	549,918	1	86
August	368,977	1	89
September	549,918	1	87
October	368,977	1	81

In the case of the single outlet, even more savings occurred by aerating the releases to 5.0 mg/1. It is noted that very little savings result when aeration to 4.0 mg/1 is compared to no aeration. This is probably because the low dissolved oxygen releases will aerate naturally to a level comparable to that achieved by aeration to 4.0 mg/1. The cost of providing the multi-level outlet can be compared with the incremental savings over the single outlet to determine which alternative is better.

In Run G, the reservoir was mixed and all releases were assumed to be 95% saturated in dissolved oxygen. Table 17 shows the decisions made in Run F and Run G. One observes that the decisions in Run F resulted in smaller release flows. This is because smaller volumes at lower temperatures have as much assimilative capacity as

TABLE 17

OPTIMUM DECISIONS FOR RUNS F AND G

Period	Releases for Run F		Releases for Run G	
	Volume Acre-ft	Temperature °F	Volume Acre-ft	Temperature °F
April	338,820	56	338,820	56
May	480,700	56	480,700	69
June	338,820	56	700,701	80
July	430,420	56	700,701	85
August	338,820	56	338,820	90
September	338,820	57	459,447	86
October	430,420	70	338,820	82

larger volumes at higher temperatures. If considerable recreation were maintained on the reservoir, a penalty cost should be imposed on excessive releases. If this were the case, then the single outlet alternative would probably appear more favorable when compared to the mixed reservoir than was found by considering only waste treatment costs.

For 1961, the costs for multi-level outlets and single outlets agreed fairly well with the costs for the same alternatives in 1958. However, the costs for the mixed reservoir were much less. This probably was due to the total available resource being greater in 1961 than in 1958. The reason the multi-level outlet costs were higher in 1961 than in 1958 is that the hypolimnion waters were colder in 1958, providing greater assimilative capacity for the summer months.

The sensitivity analysis of the parameters in the dissolved oxygen model in Chapter IV indicated considerable variance in the deficit due to k_1 when the reaeration constant, k_2, was low. Analysis of the reaeration constant for the reaches below Augusta shows that k_2 is approximately 0.15 to 0.20 day^{-1}. Therefore, it was felt that several runs should be made to observe the changes in the costs resulting from changes in the deoxygenation constant. The values for k_1 in these runs were selected as 0.25 and 0.35. The results are shown in Table 18, using the 1958 data. Comparison of the costs for Run A

TABLE 18

RESULTS OF RUNS FOR DIFFERENT k_1 VALUES

Alternative Comparable To	Costs k_1 = .25	k_1 = .35
Run A	$ 46,000	$ 91,000
Run F	77,000	157,000
Run G	102,000	145,000

and Run F in Table 18 with those in Table 14 indicate increasing savings with multi-level outlets over a single outlet with increasing k_1. This effect is not observed comparing multi-level outlets with a mixed reservoir; however, the comparison of a single outlet with the mixed reservoir indicates that the mixed reservoir becomes more favorable at higher k_1 values. Because of this sensitivity to k_1, a good estimate should be obtained before application of this methodology.

Summary of Test Case

In this chapter, the use of the methodology was demonstrated on the Savannah River. The verification of the temperature models yielded good agreement with observed data. The identification of a water quality tradeoff for the Savannah River was established, and several alternative schemes for maintaining water quality were examined. It was found that the minimum cost of the auxiliary treatment unit occurred when the hypolimnion water could be saved for release during the warmer months by use of multi-level outlets. The savings of the different schemes must be weighed against the costs of providing the alternative in order to determine the least-cost alternative. Considerable sensitivity to k_1 was demonstrated and it was stated that one must obtain a good estimate of the deoxygenation coefficient to effectively use the methodology.

VIII. SUMMARY

The objective of this study was to formulate a methodology which could be used for the evaluation of water quality alternatives in a river-reservoir system. The basis for the study was the question: if the annual reservoir inputs are known, how can the water be allocated among different time intervals in order to minimize the cost of meeting stream water quality standards? The development of the methodology required the use of a reservoir inventory model for determination of reservoir temperatures and volumes, a stream temperature model for predicting downstream temperature profiles, and a stream dissolved oxygen model for predicting the dissolved oxygen response to waste loads.

In Chapter II, the methods for predicting reservoir temperatures during thermal stratification were outlined, and the model used in this study was presented. The equilibrium temperature, determined from the reservoir heat budget, was selected as the reservoir's surface temperature. A method for representing dissolved oxygen deterioration with time was discussed and included as a segment of the reservoir model.

The stream models were developed in Chapter III. An exponential decay function was selected as the stream temperature model with the driving force being the difference between the equilibrium temperature and the stream temperature. Since research by other investigators had

shown that the parameters of Streeter-Phelps Equation were essentially random variables, the stream dissolved oxygen model was formulated as a stochastic function. By doing this, the allowable waste load which satisfied the stream standard 95% of the time could be calculated.

In Chapter IV, the concept of a water quality tradeoff was presented. Hypothetical curves showed that water at different temperatures and volumes could be substituted for one another while still maintaining water quality objectives. It was pointed out that the methodology could be applied only to situations in which the tradeoff existed.

The methodology was formulated in Chapter V as a two-dimensional dynamic programming algorithm. Because of the considerable expense of building waste treatment facilities to meet water quality problems which recur at ten year intervals, the objective function was selected as the minimization of annual treatment cost of an auxiliary unit. The two-dimensional formulation had the capability of determining the costs for several alternative schemes by the use of control variables.

A test case was presented in Chapter VII. The Savannah River in Georgia was selected as the basin to illustrate the use of the methodology. Several tests were conducted to determine the agreement of the reservoir and stream models with observed data. It was found that the epilimnion temperature provided adequate representation of the reservoir surface temperatures; the predicted reservoir release temperature agreed fairly closely with observed temperatures; and the stream temperature model yielded good agreement with observed temperature profiles. The existence of a water quality tradeoff was

established, and treatment costs for several alternatives were obtained. It was found that a multi-level outlet with release flows aerated to 5.0 mg/l yielded the minimum treatment costs; however, the savings in waste treatment must be compared with the costs of the various alternatives in the selection of the least-cost alternative. The sensitivity of the methodology to the deoxygenation coefficient was demonstrated, and it was concluded that a good estimate of the coefficient was needed to effectively use the methodology. In summary, the objective of formulating a methodology for evaluation of water quality alternatives has been accomplished. The use of the methodology has been demonstrated in a test case for several alternatives.

The user of this methodology is not restricted to minimizing only waste treatment costs. If the reservoir is used for recreation, an opportunity cost can be included in the cost function. Also, if other downstream users need the hypolimnion waters for cooling purposes, the cost to the user for warm water discharges might be added to the cost function.

A natural extension of this methodology would be to incorporate several downstream waste discharges and minimize the total annual waste treatment in the basin. In such a case, the reservoir dynamic programming technique might be superimposed on a linear programming formulation such as that of ReVelle et al.[59] Another extension would be to include several reservoirs in the methodology; however, such a problem would be very difficult, and one must remember that the water quality tradeoff must exist. Additional research could be conducted to see if Young's techniques[61] might be used to develop operating

rules for the reservoir. In doing so, a regression equation for the waste assimilative capacity could be used to reduce the computer time for determination of allowable waste loadings.

APPENDICES

A. PROBABILISTIC DEFICIT FUNCTION

The dissolved oxygen deficit is given in Equation (60) as:

$$D = \frac{K_1\ La - r}{K_2 - K_1}\left(e^{-K_1 t} - e^{-K_2 t}\right) + \left(Da\ e^{-K_2 t}\right) + \left(\frac{B+r+s}{K_2}\right)\left(1-e^{-K_2 t}\right) \quad \ldots\ (A\text{-}1)$$

The expected value and variance of D are given in Equation (63) and (65) as:

$$E(D) = D + \frac{1}{2}\left(\frac{\partial^2 D}{\partial K_1^{\,2}}\sigma_{K_1}^{\,2} + \frac{\partial^2 D}{\partial K_2^{\,2}}\sigma_{K_2}^{\,2} + \frac{\partial^2 D}{\partial r^2}\sigma_r^{\,2} + \frac{\partial^2 D}{\partial s^2}\sigma_s^{\,2}\right) \ldots\ldots\ (A\text{-}2)$$

$$Var(D) = \left(\frac{\partial D}{\partial K_1}\right)^2 \sigma_{K_1}^{\,2} + \left(\frac{\partial D}{\partial K_2}\right)^2 \sigma_{K_2}^{\,2} + \left(\frac{\partial D}{\partial r}\right)^2 \sigma_r^{\,2} + \left(\frac{\partial D}{\partial s}\right)^2 \sigma_s^{\,2} \quad \ldots\ldots\ (A\text{-}3)$$

Let $PDPK1 = \frac{\partial D}{\partial K_1}$; $PDPK12 = \frac{\partial^2 D}{\partial K_1}$

$PDPK2 = \frac{\partial D}{\partial K_2}$; $PDPK22 = \frac{\partial^2 D}{\partial K_2^{\,2}}$

$PDPR = \frac{\partial D}{\partial R}$; $PDPR2 = \frac{\partial^2 D}{\partial r^2}$

$PDPS = \frac{\partial D}{\partial S}$; $PDPS2 = \frac{\partial^2 D}{\partial s^2}$

The derivative of the product and quotient of a function is combined as follows:

$$f(x) = \frac{h(x) \cdot \ell(x)}{y(x)} ,$$

$$\frac{df(x)}{dx} = \frac{[h'(x) \cdot \ell(x) + h(x)\ \ell'(x)]\ y(x) - h(x) \cdot \ell(x)\ y'(x)}{(y(x))^2}$$

where the h'(x) denotes the first derivative with respect to x.

The partial derivatives of D are as follows:

$$PDPK1 = La\left\{\frac{[(e^{-K_1 t} - e^{-K_2 t}) - K_1 t\ e^{-K_1 t}]\ (K_2 - K_1) + K_1(e^{-K_1 t} - e^{-K_2 t})}{(K_2 - K_1)^2}\right\}$$

$$- r\left\{\frac{-t\ e^{-K_1 t}\ (K_2 - K_1) + (e^{-K_1 t} - e^{-K_2 t})}{(K_2 - K_1)^2}\right\}$$

Simplify and collecting terms yields:

$$PDPK1 = \frac{La}{K_2-K_1}\left\{\frac{(e^{-K_1t} - e^{-K_2t})}{K_2-K_1}\; K_2-K_1\; e^{-K_1t}\right\} + \frac{r}{K_2-K_1}\left\{e^{-K_1t} - \frac{e^{-K_1t} - e^{-K_2t}}{K_2-K_1}\right\}$$

..... (A-4)

$$PDPK12 = La\left\{\frac{K_2\,[(-te^{-K_1t})(K_2-K_1) + (e^{-K_1t} - e^{-K_2t})\,2\,(K_2-K_1)]}{(K_2 - K_1)^4}\right.$$

$$\left. - \left(\frac{te^{-K_1t} - K_1t\;e^{-K_1t}\,(K_2-K_1) + K_1t\,e^{-K_1t}}{(K_2 - K_1)^2}\right)\right\}$$

$$+ r\;\left\{\frac{-t^2e^{-K_1t}\,(K_2-K_1) + t\,e^{K_1t}}{(K_2 - K_1)^2}\right.$$

$$\left. - \left(\frac{-t\,e^{-K_1t}\,(K_2-K_1) + (e^{-K_1t} - e^{-K_2t})\,2\,(K_2-K_1)}{(K_2 - K_1)^4}\right)\right\}$$

Simplifying and collecting terms yields:

$$PDPK12 = La\left\{\frac{K_2}{(K_2-K_1)}\left[-t\,e^{-K_1t} + 2\,\frac{(e^{-K_1t} - e^{-K_2t})}{K_2 - K_1}\right] - \frac{K_1\,e^{-K_1t}}{K_2-K_1}\right.$$

$$\left[1 - K_1t + \frac{K_1}{K_2-K_1}\right] + r\;\frac{te^{-K_1t}}{K_2-K_1}\;\left[\frac{2}{K_2-K_1} - t\right]$$

$$\left. - 2\;\frac{(e^{-K_1t} - e^{-K_2t})}{(K_2 - K_1)^3}\right\} \qquad \text{..... (A-5)}$$

$$PDPK2 = (K_1La-r)\;\frac{te^{-K_2t}\,(K_2-K_1) - (e^{-K_1t} - e^{-K_2t})}{(K_2 - K_1)^2} - tDa\,e^{-K_2t}$$

$$+ (B+r+s)\;\frac{[K_2t\,e^{-K_2t} - (1-e^{-K_2t})]}{K_2^2} \qquad \text{..... (A-6)}$$

$$PDPK22 = (K_1 La - r) \left\{ \frac{-t^2 e^{-K_2 t} (K_2 - K_1) - te^{-K_2 t}}{(K_2 - K_1)^2} \right.$$

$$\left. - \frac{te^{-K_2 t} (K_2 - K_1)^2 - (e^{-K_1 t} - e^{-K_2 t}) 2 (K_2 - K_1)}{(K_2 - K_1)^4} \right\}$$

$$+ t^2 Dae^{-K_2 t} + (B+r+s) \left[\frac{-K_2 t^2 e^{-K_2 t} - te^{-K_2 t}}{K_2^2} - \frac{tK_2^2 e^{-K_2 t} - (1-e^{-K_2 t}) 2K_2}{K_2^4} \right]$$

Simplifying and collecting terms yields:

$$PDPK22 = \frac{(K_1 La - r)}{K_2 - K_1} te^{-K_2 t} \left(-t - \frac{2}{K_2 - K_1}\right) + 2 \frac{(e^{-K_1 t} - e^{-K_2 t})}{(K_2 - K_1)^2}$$

$$+ t^2 Dae^{-K_2 t} + (B+r+s) \left[te^{-K_2 t} \left(-\frac{2}{K_2} - t\right) + 2 \frac{(1-e^{-K_2 t})}{K_2^2} \right] \quad \text{..... (A-7)}$$

$$PDPR = \frac{1}{K_2} (1-e^{-K_2 t}) - \frac{1}{K_2 - K_1} (e^{-K_1 t} - e^{-K_2 t}) \quad \text{..... (A-8)}$$

$$PDPR2 = 0 \quad \text{..... (A-9)}$$

$$PDPS = \frac{1}{K_2} (1 - e^{-K_2 t}) \quad \text{..... (A-10)}$$

$$PDPS2 = 0 \quad \text{..... (A-11)}$$

Substituting Equations (A-3) to (A-10) into Equation (A-2) and (A-3) give the following expressions for E(D) and Var(D):

$$E(D) = D + \frac{1}{2} [PDPK12 (\sigma_{K_1}^2) + PDPK22 (\sigma_{K_2}^2)] \quad \text{..... (A-12)}$$

$$Var(D) = (PDPK1)^2 \sigma_{K_1}^2 + (PDPK2)^2 \sigma_{K_2}^2 + (PDPR)^2 \sigma_r^2 + (PDPS)^2 \sigma_s^2$$

$$\text{..... (A-13)}$$

The same procedure is applied to the equation for L, Equation (59) to find E(L) and Var(L).

$$L = (La - \frac{r}{K_1} e^{-K_1 t} + \frac{r}{K_1} \quad \text{..... (A-14)}$$

$$E(L) = L + \frac{1}{2}\left[\frac{\partial^2 L}{\partial K_1^2}\,\sigma_{K_1}^2 + \frac{\partial^2 L}{\partial r^2}\,\sigma_r^2\right] \quad \text{..... (A-15)}$$

$$\mathrm{Var}(L) = \left(\frac{\partial L}{\partial K_1}\right)^2 \sigma_{K_1}^2 + \left(\frac{\partial L}{\partial r}\right) \sigma_r^2 \quad \text{..... (A-16)}$$

Let $\quad PLPK1 = \frac{\partial L}{\partial K_1}$; $\quad PLPK12 = \frac{\partial^2 L}{\partial K_1^2}$

$\quad PLPR = \frac{\partial L}{\partial r}$; $\quad PLPR2 = \frac{\partial^2 L}{\partial r^2}$

Performing the partial derivatives yields:

$$PLPK1 = -K_1\, Lae^{-K_1 t} + r\left\{ e^{-K_1 t} - \frac{1-e^{-K_1 t}}{K_1} \right\} \quad \text{..... (A-17)}$$

$$PLPK12 = Lae^{-K_1 t}\,(K_1^2-1) + r\left(e^{-K_1 t}\left[\frac{2}{K_1^3} - K_1 - \frac{1}{K_1}\right] - \frac{1}{K_1^3}\right) \quad \text{..... (A-18)}$$

$$PLPR = \frac{1}{K_1}\,(1 - e^{-K_1 t}) \quad \text{..... (A-19)}$$

$$PLPR2 = 0 \quad \text{..... (A-20)}$$

Substituting Equation (A-17) to (A-20) in Equations (A-15) and (A-16) yields:

$$E(L) = L + \frac{1}{2}\,(PLPK12\,(\sigma_{K_1}^2)) \quad \text{..... (A-21)}$$

$$\mathrm{Var}(L) = (PLPK1)^2\,\sigma_{K_1}^2 + (PLPR)^2\,\sigma_r^2 \quad \text{..... (A-22)}$$

Equation (A-12), (A-13), (A-21), and (A-22) can be used for deterministic Da and La. When the equations are used to determine the deficit at the ends of each reach, additional terms must be included because of the stochastic nature of Da and La. These are given below:

Let $\quad PDPDa = \frac{\partial D}{\partial Da}$; $\quad PDPDa2 = \frac{\partial^2 D}{\partial Da^2}$

$\quad PDPLa = \frac{\partial D}{\partial La}$; $\quad PDPLa2 = \frac{\partial^2 D}{\partial La^2}$

$$PLPLa = \frac{L}{La} \quad ; \quad PLPLa2 = \frac{L}{La}$$

$$PDPDa = e^{-K_2 t} \quad \text{..... (A-23)}$$

$$PDPDa2 = 0 \quad \text{..... (A-24)}$$

$$PDPLa = \frac{K_1}{K_2 - K_1} (e^{-K_1 t} - e^{-K_2 t}) \quad \text{..... (A-25)}$$

$$PDPLa2 = 0 \quad \text{..... (A-26)}$$

$$PLPLa = e^{-K_1 t} \quad \text{..... (A-27)}$$

$$PLPLa2 = 0 \quad \text{..... (A-28)}$$

Letting the subscript i refer to the reach number and including the Equations (A-23) to (A-28), the recursive application of the stochastic equations for the deficit at the end of reach i is:

$$E(D)_i = D_i + \frac{1}{2} [PDPK12 \ (\sigma_{K_1}{}^2) + PDPK22 \ (\sigma_{K_2}{}^2)] \quad \text{.....(A-29)}$$

$$Var(D)_i = (PDPK1)^2 \sigma_{K_1}{}^2 + (PDPK2)^2 \ \sigma_{K_2}{}^2 + (PDPR)^2 \ \sigma_r{}^2 + (PDPS)^2 \ \sigma_s{}^2$$

$$+ PDPDa \ (Var(D)_{i-1}) + PDPLa \ (Var(L)_{i-1}) \quad \text{..... (A-30)}$$

$$E(L)_i = L_i + \frac{1}{2} [\partial LPK12 \ (\sigma_{K_1}{}^2)] \quad \text{..... (A-31)}$$

$$Var(L)_i = PLPK1 \ (\sigma_{K_1}{}^2) + PLPR \ (\sigma_r{}^2) + PLPLa \ (Var(L)_{i-1}) \quad \text{..... (A-32)}$$

The expressions for the partial derivatives have been coded in computer language. The equations in the Appendix are in the listing of subroutine DODEF in Appendix B.

B. PROGRAM LISTING

```
C      WATER QUALITY DYNAMIC PROGRAMMING ALGORITHM
       DIMENSION X(15),T(15),STRDO(15),STRBOD(15),FK(21,3),FK
                                                        M1(21,3),
      1VOL(8,21,3,3),QUAL(8,21,3,3),BOD1(8,21,3,3),DO1(8,21,3
                                                        ,3),
      2SHORTW(15),TEMPAI(15),WIND(15),AIRVAP(15),CC(15),
      3EPIDEP(15),AKA1(15),AK11(12),RDOC(8,3),
      4FLO(15,12),BODIN(12),DOIN(12),VEMEAN(12),DMEAN(12),TRA
                                                        TI(12),TOTRA
      5D(15),ALPHAV(12),ALPHAD(12),BETAV(12),BETAD(12),
      6DISREA(12),TEMIN(12),WATEMP(12),BEGTEM(12),AVGTEM(12),
                                                        AKA2(12)
       COMMON GAMMA1,GAMMA2,VOL,QUAL,DO1,BOD1,FLO,BODIN,DOIN,
                                                        TEMIN,
      1DOMIX,BODMIX,WIND,CC,SHORTW,VEMEAN,DMEAN,TRATI,X,T,TOT
                                                        RAD,
      2ALPHAV,ALPHAD,BETAV,BETAD,DISREA,WATEMP,BEGTEM,AVGTEM,
                                                        AKA2,
      3AK11,STRDO,STRBOD,FACTOR,EPIDEP,AKA1,TEMPAI,AIRVAP
       COMMON NREACH,N,NWL,NL,PMIN,SMIN,PMAX,SMAX,RDOC
       INTEGER GAMMA1,GAMMA2,VOL,QUAL,X,T,SO,SMIN,PMIN,TO,PMA
                                                        X,EPIDEP,
      1SMAX
       INTEGER STOR1,STOR2,STOR3,TEM1,TEM2,TEM3
       DIMENSION DEC1(8,21,3),DEC2(8,21,3),DEC3(8,21,3),DEC4(
                                                        8,21,3),
      1DEC5(8,21,3),DEC6(8,21,3),DEC7(8,21,3),DEC8(8,21,3),DE
                                                        C9(8,21,3),
      2DEC10(8,21,3)
       DATA FK/63*0.0/,FKM1/63*0.0/
C      BEGIN READING DATA
       READ(5,5000) KEY1,IK8,IAERAT,DORELS
 5000  FORMAT(3I5,F5.0)
       READ(5,5001) NREACH,N,NWL,NL,SMIN,SMAX,DMINAL,WINCRE,F
                                                        ACTOR,NW
 5001  FORMAT(4I5,2I10,3F10.0,I5)
       READ(5,5002)(ALPHAV(I),ALPHAD(I),BETAV(I),BETAD(I),DIS
                                                        REA(I),I=1,N
      1REACH)
 5002  FORMAT(5F10.5)
       READ(5,5003)  RMEAN,SMEAN,RSTD,SSTD,AK1STD,AK2STD
 5003  FORMAT(6F10.0)
       READ(5,5004) ((RDOC(I,J),I=1,8),J=1,3)
 5004  FORMAT(8F5.0)
       READ(5,5005) (EPIDEP(I),I=1,N)
 5005  FORMAT(8I10)
       READ(5,5006) (AKA1(I),I=1,N)
 5006  FORMAT(8F10.0)
C      READ ANNUAL DATA
     1 READ(5,5007) SO,TO,DOI,BODI
 5007  FORMAT(2I10,2F10.0)
       READ(5,5008)(X(I),T(I),STRDO(I),STRBOD(I),I=1,N)
 5008  FORMAT(2I10,2F10.2)
```

```
      READ(5,5009)((FLO(I,J),J=1,N),I=1,NREACH)
 5009 FORMAT(7F6.0)
      READ(5,5010)(TEMPAI(I),AIRVAP(I),WIND(I),CC(I),SHORTW(
                                                    I),I=1,N)
 5010 FORMAT(       10X,F10.8,F10.8,F10.9,F10.9,F10.7)
      WRITE(6,6005)N,NWL,SO,NL,TO,BOD1,DOI,FACTOR
 6005 FORMAT(1H0,5I10,3F10.5)
      WRITE(6,6006)(X(I),T(I),STRDO(I),STRBOD(I),EPIDEP(I),S
                                                    HORTW(I),
     1TEMPAI(I),WIND(I),AIRVAP(I),CC(I),I=1,N)
 6006 FORMAT(1H0,2I10,2F10.2,I10,5F10.2)
      WRITE(6,6010) (ALPHAV(I),ALPHAD(I),BETAV(I),BETAD(I),D
                                                    ISREA(I),
     1I=1,NREACH)
 6010 FORMAT(1H0,5F10.6)
      WRITE(6,6011)NREACH,DMINAL,NW,WINCRE
 6011 FORMAT(1H0,'NREACH =',I5,F10.2,I10,F10.2)
      WRITE(6,6031) RMEAN,SMEAN,RSTD,SSTD,AK1STD,AK2STD
 6031 FORMAT(1H0,6F10.3)
      WRITE(6,6032) ((RDOC(I,J),J=1,3),I=1,8)
 6032 FORMAT(1H0,3F10.2)
      MNUM = 0
C     DETERMINE PMIN,PMAX,INCRE,  ALSO CALC TOTAL RADIATION,
                                                    TOTRAD.
      CALL RADCAL(SO,INCRE,ITCAVR)
C     INITIALIZE STATE VARIABLES
      K = 1
      CALL EQUIL(K,E,AK2)
      STAGE = DEPTH(SO)
      STAGE1 = STAGE-EPIDEP(1)
      VOLUME = CAPAC(STAGE)-CAPAC(STAGE1)
      VOLUM1 = SO-VOLUME
      IF(NL.EQ.1)  VOLUME = SO
      DO 10 I=1,NWL
      DO 10 J=1,NL
      FKM1(I,J) = 0.
      DO 10 K=1,NL
      IF(K.EQ.1) VOL(1,I,J,K) = VOLUME
      IF(K.EQ.2) VOL(1,I,J,K) = 0
      IF(K.EQ.3) VOL(1,I,J,K) = VOLUM1
      QUAL(1,I,J,K) = TO
      IF(K.EQ.1.AND.NL.NE.1) QUAL(1,1,J,K) = E
      BOD1(1,I,J,K) = BODI
      DO1(1,I,J,K) = DOI
   10 CONTINUE
      WRITE(6,6300) (VOL(1,1,1,I),QUAL(1,1,1,I),I=1,NL)
 6300 FORMAT(1H0,2I10)
C     BEGIN DYNAMIC PROGRAMMING--K IS STAGE NO.
      JM1 = NWL
      K = 1
   33 WRITE(6,6012)TOTRAD(K)
 6012 FORMAT(1H0,'TOTAL RADIATION FOR KTH PERIOD IS',F10.2)
      IF(K.NE.1)  CALL EQUIL(K,E,AK2)
```

```
      T2= (E-32.)*5./9.
      SAT = (14.652-.41022*T2+.007991*T2*T2-.C00077774*T2**3
                                                        .)*.85
      DO 60 I=1,NREACH
      BODIN(I) = 2.
      IF(I.EC.NW)  BODIN(I) = 0.
      DOIN(I) = SAT
      IF(I.EQ.NW)  DOIN(NW) = 2.0
   60 TEMIN(I) = T(K)
      WRITE(6,6013)(TEMIN(J),FLO(J,K) ,BODIN(J),DOIN(J),J=1,
                                                        NREACH)
 6013 FORMAT(1HO,4F10.2)
      MAXALL = K*PMAX
      ITEST = ITOAVR-(N-K)*PMIN
      IF(MAXALL.GT.ITEST)  MAXALL = ITEST
      JMAX =(MAXALL-K*PMIN)/INCRE+1
      J1 = 1
   31 K1 = 1
      IF(IK8.GT.0)  K1 = 3
   27 BETA = 1.0E10
      GAMMA1 = 0
      GAMMA2 = 0
      IGAM11 = 0
      IGAM22 = 0
      IGAM33 = 0
      IK11 = 0
      IK22 = 0
      IK33 = 0
      MIX1 = 0
      IP3 = PMIN
      J2 = 1
   25 K2 = 1
      IF(IK8.GT.0)  K2 = 3
      J3 = J1-J2+1
      IF(J3.GT.JM1)  GO TO 24
      IF(IP3.GT.PMAX)  GO TO 26
C     DETERMINE QUALITY OF RESERVOIR RELEASE.
   23 CALL DISCHA(K,J3,K1,K2,MIX1,IGAM11,IGAM22,IGAM33,IK11,
                                                        IK22,IK33,
     1IP3)
      WRITE(6,6008)GAMMA1,GAMMA2,MIX1,DOMIX,BODMIX
 6008 FORMAT(' ','GAMMA1 =',I10,'  GAMMA2 =',I10,'  MIX1 =',
                                                        I10,2F10.2)
      IF(GAMMA1.GT.0)  GO TO 35
      ALAWLO = 0.
      GO TO 36
   35 IF(DOMIX.LT.DMINAL.AND.IAERAT.GT.0)  DOMIX = DOREQS
C     DETERMINE ALLOWABLE WASTE LOAD.
      CALL ALLOWA(K,NW,WINCRE,DMINAL,ALAWLO,MNUM,E,AK2,RMEAN
                                                        ,SMEAN,RSTD,
     1SSTD,AK1STD,AK2STD)
C     DETERMINE AUXILIARY UNIT COST.
   36 CALL AUXCOS(ALAWLO,COST2,TEMIN(NW))
```

```
      WRITE(6,6002) ALAMLC,COST2
 6002 FORMAT(' ',10X,'ALAMLC =',F20.2,'     COST2 = ',F20.2)
C     EQUATION (82).
      ALPHA = COST2 + FKM1(J3,K2)
      IF(ALPHA-BETA)21,22,22
   21 BETA = ALPHA
      IK1 = IK11
      IK2 = IK22
      IK3 = IK33
      MIX = MIX1
      IGAM1 = IGAM11
      IGAM2 = IGAM22
      IGAM3 = IGAM33
      IQUAN1 = GAMMA1
      IQUAL1 = GAMMA2
      JSAVE = J3
      KSAVE = K2
   22 MNUM = 2
      K2 = K2+1
      IF(K.EQ.1)  K2 = NL+1
      IF(K2-NL)23,23,24
   24 IP3 = IP3+INCRE
      J2 = J2+1
      IF(J2-J1)25,25,26
   26 WRITE(6,6020)IQUAN1,IQUAL1,IK1,IK2,IK3,IGAM1,IGAM2,IGA
                                                    M3,JSAVE,
     1KSAVE
 6020 FORMAT(' ',10I10)
C     TRANSFORM RESERVOIR ACCORDING TO DECISIONS.
      CALL TRANSK(K,J1,K1,MIX,IK1,IK2,IK3,IGAM1,IGAM2,IGAM3,
                                                    IQUAN1,
     1JSAVE,KSAVE,E,KEY1)
      FK(J1,K1) = BETA
      DEC1(K,J1,K1) = IQUAN1
      DEC2(K,J1,K1) = IQUAL1
      DEC3(K,J1,K1) = IK1
      DEC4(K,J1,K1) = IK2
      DEC5(K,J1,K1) = IK3
      DEC6(K,J1,K1) = IGAM1
      DEC7(K,J1,K1) = IGAM2
      DEC8(K,J1,K1) = IGAM3
      DEC9(K,J1,K1) = JSAVE
      DEC10(K,J1,K1) = KSAVE
      K1 = K1+1
      IF(K1-NL)27,27,28
   28 J1 = J1+1
      IF(J1.LE.JMAX)  GO TO 31
C     BLOCK TRANSFER
   32 DO 40 I=1,JMAX
      DO 40 J=1,NL
   40 FKM1(I,J) = FK(I,J)
      WRITE(6,6003)((FKM1(I,J),J=1,NL),I=1,JMAX)
 6003 FORMAT(1H0,10X,3F20.2/)
```

```
      JM1 = JMAX
      K = K+1
      IF(K-N)33,33,34
C     DETERMINE OPTIMAL DECISIONS
   34 WRITE(6,6000)
 6000 FORMAT(1H1,59X,'QUANTITY WITHDRAWN'/'  PERIOD  TOTAL D
                                                ISCHARGE  TE
     IMPERATURE  LAYER WITHDRAWN  IN ACRE-FEET FROM LAYER',5
                                                X,'JSAVE  KS
     3AVE'/
     213X,'ACRE-FEET       DEG. F',11X,'FROM',9X,'1',6X,'2',
                                                7X,'3',//)
      BETA = 1.0E10
      JINDEX = 1
      IF(IK8.GT.0)  JINDEX = 3
      DO 80 J = JINDEX,NL
      DO 80 I=1,JMAX
      TEST = FK(I,J)
      IF(TEST-BETA)81,80,80
   81 BETA = TEST
      ISAVE1 = I
      JSAVE1 = J
   80 CONTINUE
      J1 = ISAVE1
      K1 = JSAVE1
      DO 50 I=1,N
      K = N-I+1
      IQUAN1 = DEC1(K,J1,K1)
      IQUAL1 = DEC2(K,J1,K1)
      IK1 = DEC3(K,J1,K1)
      IK2 = DEC4(K,J1,K1)
      IK3 = DEC5(K,J1,K1)
      IGAM1 = DEC6(K,J1,K1)
      IGAM2 = DEC7(K,J1,K1)
      IGAM3 = DEC8(K,J1,K1)
      JSAVE = DEC9(K,J1,K1)
      KSAVE = DEC10(K,J1,K1)
      J1 = JSAVE
      K1 = KSAVE
      WRITE(6,6001)K,IQUAN1,IQUAL1,IK1,IK2,IK3,IGAM1,IGAM2,I
                                                GAM3,JSAVE,
     1KSAVE
 6001 FORMAT(1HO,I6,2I14,I9,2I5,I10,I8,I8,I8,I7)
   50 CONTINUE
      WRITE(6,6030)
 6030 FORMAT(1H1,22X,'STORAGE IN LAYER',22X,'TEMPERATURE IN
                                                LAYER',//
     1'  PERIOD       ONE        TWO       THREE
                                                  ONE
     1 TWO       THREE')
      J1 = ISAVE1
      K1 = JSAVE1
      DO 70 I=1,N
```

```
      K = N-I+1
      STOR1 = VOL(K,J1,K1,1)
      STOR2 = VOL(K,J1,K1,2)
      STOR3 = VOL(K,J1,K1,3)
      TEM1 = QUAL(K,J1,K1,1)
      TEM2 = QUAL(K,J1,K1,2)
      TEM3 = QUAL(K,J1,K1,3)
      JSAVE = DEC9(K,J1,K1)
      KSAVE = DEC10(K,J1,K1)
      J1 = JSAVE
      K1 = KSAVE
      IF(NL.EQ.1)  GO TO 6042
      WRITE(6,6040) K,STOR1,STOR2,STOR3,TEM1,TEM2,TEM3
 6040 FORMAT(1H0,I6,I14,2I10,I15,2I10)
      GO TO 70
 6042 WRITE(6,6041) K,STOR1,TEM1
 6041 FORMAT(1H0,I6,I14,20X,I15)
   70 CONTINUE
C     READ ANNUAL DATA.
      GO TO 1
      END
      SUBROUTINE AUXCOS(ALWALC,COST2,TEMP)
C     ROUTINE FOR DETERMINING THE COST OF AUXILIARY WASTE TR
                                                    EATMENT.
      BOD = 99994.5984*(.02*(TEMP-32.)*5./9.+.6)
      TEST = BOD-ALWALO
      IF(TEST.GT.0.5)  GO TO 1
      COST2 = 0.
      RETURN
    1 VOLUME = TEST*75./(BOD*.75)
      IF(VOLUME.GT.75.)  GO TO 2
      COS = .01*3./(VOLUME**.0311)
      COST2 = COS*VOLUME*1000.*30.4
      RETURN
C     WHEN THE UNIT CANNOT REMOVE THE NECESSARY BOD THEN COS
                                                    T2 IS MADE
C     A LARGE POSITIVE NUMBER.
    2 COST2 = 1.0E06
      RETURN
      END
      SUBROUTINE RADCAL(SC,INCRE,ITOAVR)
C     DETERMINATION OF PMIN,PMAX,INCRE,AND TOTRAD.
      DIMENSION X(15),T(15),STRDO(15),STRBOD(15),FK(21,3),FK
                                                    M1(21,3),
     1VOL(8,21,3,3),QUAL(8,21,3,3),BOD1(8,21,3,3),DO1(8,21,3
                                                    ,3),
     2SHORTW(15),TEMPAI(15),WIND(15),AIRVAP(15),CC(15),
     3EPIDEP(15),AKA1(15),AK11(12),RDOC(8,3),
     4FLO(15,12),BODIN(12),DOIN(12),VEMEAN(12),DMEAN(12),TRA
                                                    TI(12),TOTRA
     5D(15),ALPHAV(12),ALPHAD(12),BETAV(12),BETAD(12),
     6DISREA(12),TEMIN(12),WATEMP(12),BEGTEM(12),AVGTEM(12),
                                                    AKA2(12)
```

```
      COMMON GAMMA1,GAMMA2,VOL,QUAL,DC1,BCD1,FLO,BODIN,DOIN,
                                                        TEMIN,
     IDOMIX,BODMIX,WIND,CC,SHORTW,VEMEAN,DMEAN,TRATI,X,T,TOT
                                                        RAD,
     2ALPHAV,ALPHAD,BETAV,BETAD,DISREA,WATEMP,BEGTEM,AVGTEM,
                                                        AKA2,
     3AK11,STRDO,STRBOD,FACTOR,EPIDEP,AKA1,TEMPAI,AIRVAP
      COMMON NREACH,N,NWL,NL,PMIN,SMIN,PMAX,SMAX,RDOC
      INTEGER GAMMA1,GAMMA2,VOL,QUAL,X,T,SO,SMIN,PMIN,TO,PMA
                                                        X,EPIDEP,
     ISMAX
C     MINAQ=MIN. ALLOWABLE RELEASE.  MAXAQ=MAX. ALLOWABLE RE
                                                        LEASE.
C     ISFIN=MIN. STORAGE IN OCTOBER.  EVAP=AVERAGE EVAPORATI
                                                        ON TOTAL.
      DATA MINAQ,MAXAQ,ISFIN,EVAP/241255,904704,2350000,   0
                                                        ./
      ISUM = 0
      DO 20 I=1,N
      SHORTW(I) = SHORTW(I)*3.68
      BETA = CLODCO(AIRVAP(I),CC(I))
      TOTRAD(I) = .95*SHORTW(I)+(BETA*4.15E-08)*(460.+TEMPAI
                                                        (I))**4
      X(I) = X(I)/FACTOR
      ISUM = ISUM+X(I)
   20 CONTINUE
      ITOAVR = ISUM+SO-ISFIN-EVAP/FACTOR
      AVG = ITOAVR/N
      IF(NWL-2)1,2,3
    1 PMIN = AVG
      PMAX = AVG
      INCRE = AVG
      RETURN
    2 PMIN = AVG-1500/FACTOR
      PMAX = AVG+1500/FACTOR
      GO TO 30
    3 IF(NWL-4)4,5,6
    4 PMIN = AVG-1500/FACTOR
      PMAX = AVG+1500/FACTOR
      GO TO 30
    5 PMIN = AVG-2000/FACTOR
      PMAX = AVG+4000/FACTOR
      GO TO 30
    6 PMIN = AVG-3000/FACTOR
      PMAX = AVG+6000/FACTOR
   30 IF(PMAX.GT.MAXAQ) PMAX = MAXAQ
      IF(PMIN.LT.MINAQ) PMIN = MINAQ
      INCRE = (PMAX-PMIN)/(NWL-1)
      RETURN
      END
      SUBROUTINE EQUIL(K,E,AK2)
C     ROUTINE FOR CALCULATING EQUILIBRIUM TEMPERATURE BY EQU
                                                        ATION (28)
```

```
C      AND TABLE 1.
       DIMENSION CB(6,3)
       DIMENSION X(15),T(15),STRDO(15),STRBOD(15),FK(21,3),FK
                                                          M1(21,3),
      1VOL(8,21,3,3),QUAL(8,21,3,3),BOD1(8,21,3,3),DO1(8,21,3
                                                          ,3),
      2SHORTW(15),TEMPAI(15),WIND(15),AIRVAP(15),CC(15),
      3EPIDEP(15),AKA1(15),AK11(12),RDOC(8,3),
      4FLO(15,12),BODIN(12),DOIN(12),VEMEAN(12),DMEAN(12),TRA
                                                          T1(12),TOTRA
      5D(15),ALPHAV(12),ALPHAD(12),BETAV(12),BETAD(12),
      6DISREA(12),TEMIN(12),WATEMP(12),BEGTEM(12),AVGTEM(12),
                                                          AKA2(12)
       COMMON GAMMA1,GAMMA2,VOL,QUAL,DO1,BOD1,FLO,BODIN,DOIN,
                                                          TEMIN,
      1DOMIX,BODMIX,WIND,CC,SHORTW,VEMEAN,DMEAN,TRAT1,X,T,TOT
                                                          RAD,
      2ALPHAV,ALPHAD,BETAV,BETAD,DISREA,WATEMP,BEGTEM,AVGTEM,
                                                          AKA2,
      3AK11,STRDO,STRBOD,FACTOR,EPIDEP,AKA1,TEMPAI,AIRVAP
       COMMON NREACH,N,NML,NL,PMIN,SMIN,PMAX,SMAX,RDOC
       INTEGER GAMMA1,GAMMA2,VOL,QUAL,X,T,SO,SMIN,PMIN,TO,PMA
                                                          X,EPIDEP,
      1SMAX
       DATA CB/45.,55.,65.,75.,85.,95.,.291,.405,.5553,.6667,
                                                          .990,1.289,
      1-5.47,-11.22,-20.15,-27.80,-53.33,-89.28/
       AHR = TOTRAD(K)
       TA = TEMPAI(K)
       EA = AIRVAP(K)
       W = WIND(K)
        IF(TA.LT.50.)GO TO 5
        IF(TA.LT.60.)GO TO 6
        IF(TA.LT.70.)GO TO 7
        IF(TA.LT.80.)GO TO 8
        IF(TA.LT.90.)GO TO 9
        ISAVE = 6
        GO TO 10
     5  ISAVE = 1
        GO TO 10
     6  ISAVE = 2
        GO TO 10
     7  ISAVE = 3
        GO TO 10
     8  ISAVE = 4
        GO TO 10
     9  ISAVE = 5
    10   E1 = CB(ISAVE,1)
    17  B1 = 0.26+CB(ISAVE,2)
        AK2 = 15.7+B1*(73.+7.3*W)
         B2 = (AHR-1801.)/AK2
         B3 = (AK2-15.7)/AK2
    11   B4 = 1.+E1*.051/AK2
```

```
      E = (B2+B3*((.26*TA/B1)+(EA-CB(ISAVE,3))/B1))/B4
     E2 = ABS(E-E1)
      IF(E2-1.)16,16,18
   18  N2= E
     N1 = N2/10-3
     IF(N1-ISAVE)12,13,14
   12  ISAVE = ISAVE-1
      E1 = E
     GO TO 17
   14  ISAVE = ISAVE +1
      E1 = E
     GO TO 17
   13  E1 = E
      GO TO 11
   16 CONTINUE
     RETURN
      END
     SUBROUTINE DISCHA(K,J3,K1,K2,MIX1,IGAM11,IGAM22,IGAM33
                                                 ,IK11,IK22,
    1IK33,IP3)
C      ROUTINE TO DETERMINE QUALITY OF RELEASE.
      DIMENSION X(15),T(15),STRDO(15),STRBOD(15),FK(21,3),FK
                                                 M1(21,3),
    1VOL(8,21,3,3),QUAL(8,21,3,3),BOD1(8,21,3,3),DO1(8,21,3
                                                 ,3),
    2SHORTW(15),TEMPAI(15),WIND(15),AIRVAP(15),CC(15),
    3EPIDEP(15),AKA1(15),AK11(12),RDOC(8,3),
    4FLO(15,12),BODIN(12),DOIN(12),VEMEAN(12),DMEAN(12),TRA
                                                 T1(12),TOTRA
    5D(15),ALPHAV(12),ALPHAD(12),BETAV(12),BETAD(12),
    6DISREA(12),TEMIN(12),WATEMP(12),BEGTEM(12),AVGTEM(12),
                                                 AKA2(12)
      COMMON GAMMA1,GAMMA2,VOL,QUAL,DO1,BOD1,FLO,BODIN,DOIN,
                                                 TEMIN,
    1DOMIX,BODMIX,WIND,CC,SHORTW,VEMEAN,DMEAN,TRAT1,X,T,TOT
                                                 RAD,
    2ALPHAV,ALPHAD,BETAV,BETAD,DISREA,WATEMP,BEGTEM,AVGTEM,
                                                 AKA2,
    3AK11,STRDO,STRBOD,FACTOR,EPIDEP,AKA1,TEMPAI,AIRVAP
      COMMON NREACH,N,NWL,NL,PMIN,SMIN,PMAX,SMAX,RDOC
      INTEGER GAMMA1,GAMMA2,VOL,QUAL,X,T,SO,SMIN,PMIN,TO,PMA
                                                 X,EPIDEP,
    1SMAX
C      IF THE SELECTED LAYER DOES NOT HAVE REQUIRED VOLUME,TH
                                                 EN THE
C      ADDITIONAL VOLUME IS TAKEN FROM ANOTHER LAYER AND ALL
                                                 QUALITY
C      PARAMETERS ARE MIXED.
      IP4 = IP3
      IP2 = 0
C      TEST TO SEE IF THE MAXIMUM ALLOWABLE STORAGE IS EXCEED
                                                 ED, IF SO
C      RELEASE ENOUGH TO SATISFY THIS CONSTRAINT.
```

```
      ITOVOL = 0
      DO 100 I=1,NL
  100 ITOVOL = ITOVOL+VOL(K,J3,K2,I)
      IF(ITOVOL+X(K)-SMAX)101,101,102
  102 IPMIN = ITOVOL-SMAX+X(K)
      IF(IPMIN-IP3 )101,101,103
  103 IP3 = IPMIN
  101 IF((ITOVOL-IP3)-SMIN)20,21,21
   20 N1= IP3/PMIN
   26 IF(N1)22,22,23
   23 IP2 = (N1-1)*PMIN
      IF((ITOVOL-IP2)-SMIN)24,25,25
   24 N1 = N1-1
      GO TO 26
   25 IP4 = IP3
      IP3 = IP2
      GO TO 21
C     MINIMUM ALLOWABLE STORAGE CONSTRAINT MUST BE SATISFIED

   22 GAMMA1 = 0
      MIX1 = 0
      IK11 = 0
      IK22 = 0
      IK33 = 0
      IGAM11 = 0
      IGAM22 = C
      IGAM33 = 0
      GAMMA2 = 0
      RETURN
C     IF VOLUME IN LAYER IS SUFFICIENT, THEN IK11 = IK1 AND
                                                MIX1=0.
   21 GAMMA1 =VOL(K,J3,K2,K1)
      IF(GAMMA1-IP3)2,1,1
    1 GAMMA1 = IP3
      GAMMA2 = QUAL(K,J3,K2,K1)
      MIX1 = 0
      IK11 = K1
      COMIX =DO1(K,J3,K2,K1)
      BODMIX = BOD1(K,J3,K2,K1)
      IK22 = C
      IK33 = 0
      IGAM11 = 0
      IGAM22 = 0
      IGAM33 = 0
      IF(K1-2)31,32,33
   31 IGAM11 = GAMMA1
      GO TO 34
   32 IGAM22 = GAMMA1
      GO TO 34
   33 IGAM33 = GAMMA1
   34 GO TO 28
    2 IF(K1-2)3,4,5
    3 IGAM11 = GAMMA1
```

```
      IGAM22 = VOL(K,J3,K2,K1+1)
      IF((IGAM11+IGAM22)-IP3)6,7,7
C     MUST MIX TWO UPPER LAYERS.
    7 IGAM22 = IP3-IGAM11
      MIX1 = 1
      IK11 = K1
      IK22 = K1+1
      IK33 = 0
      GAMMA1 = IP3
      IGAM33 = 0
      BODMIX = (BOD1(K,J3,K2,K1)*IGAM11+BOD1(K,J3,K2,K1+1)*I
                                                    GAM22)/IP3
      DOMIX = (DO1(K,J3,K2,K1)*IGAM11+DO1(K,J3,K2,K1+1)*IGAM
                                                    22)/IP3
      GAMMA2 = (QUAL(K,J3,K2,K1)*IGAM11+QUAL(K,J3,K2,K1+1)*I
                                                    GAM22)/IP3
      GO TO 28
C     MUST MIX ALL THREE LAYERS.
    6 IGAM33 = VOL(K,J3,K2,3)
      IF((IGAM11+IGAM22+IGAM33)-IP3)8,9,9
    9 IGAM33 = IP3-(IGAM11+IGAM22)
      MIX1 = 2
      IK11 = 1
      IK22 = 2
      IK33 = 3
      GAMMA1 = IP3
      BODMIX = (BOD1(K,J3,K2,1)*IGAM11+BOD1(K,J3,K2,2)*IGAM2
                                                    2+BOD1(K,J3,
     1K2,3)*IGAM33)/IP3
      DOMIX = (DO1(K,J3,K2,1)*IGAM11+DO1(K,J3,K2,2)*IGAM22+D
                                                    O1(K,J3,K2,3
     1)*IGAM33)/IP3
      GAMMA2 = (QUAL(K,J3,K2,1)*IGAM11+QUAL(K,J3,K2,2)*IGAM2
                                                    2+QUAL(K,J3,
     1K2,3)*IGAM33)/IP3
      GO TO 28
C     RELEASE EXCEEDS ALL STORAGE--SOMETHING IS WRONG.
    8 WRITE(6,6000)
 6000 FORMAT(1H1,10X,'RESERVOIR DISCHARGE WILL EMPTY STORAGE
                                                    ')
      CALL EXIT
C     MIX LOWER TWO LAYERS.
    4 IGAM11 = GAMMA1
      IGAM22 = VOL(K,J3,K2,3)
      IF((IGAM11+IGAM22)-IP3)10,11,11
   11 GO TO 7
   10 IGAM33 = VOL(K,J3,K2,1)
      IF((IGAM11+IGAM22+IGAM33)-IP3)8,12,12
   12 IGAM33 = IGAM22
      IGAM22 = IGAM11
      IGAM11 = IP3-(IGAM22+IGAM33)
      MIX1 = 2
      IK11 = 1
```

```
      IK22 = 2
      IK33 = 3
      GAMMA1 = IP3
      BODMIX = (BOD1(K,J3,K2,1)*IGAM11+BOD1(K,J3,K2,2)*IGAM2
                                                    2+BOD1(K,J3,
     1K2,3)*IGAM33)/IP3
      DOMIX = (DO1(K,J3,K2,1)*IGAM11+DO1(K,J3,K2,2)*IGAM22+D
                                                    O1(K,J3,K2,3
     1)*IGAM33)/IP3
      GAMMA2 = (QUAL(K,J3,K2,1)*IGAM11+QUAL(K,J3,K2,2)*IGAM2
                                                    2+QUAL(K,J3,
     1K2,3)*IGAM33)/IP3
      GO TO 28
    5 IGAM11 = GAMMA1
      IGAM22 = VOL(K,J3,K2,2)
      IF((IGAM11+IGAM22)-IP3)13,14,14
   14 IGAM11 = IP3-IGAM11
      IGAM22 = IP3-IGAM11
      IGAM33 = 0
      MIX1 = 1
      IK11 = 2
      IK22 = 3
      IK33 = 0
      GAMMA1 = IP3
      BODMIX = (BOD1(K,J3,K2,2)*IGAM11+BOD1(K,J3,K2,3)*IGAM2
                                                    2)/IP3
      DOMIX = (DO1(K,J3,K2,2)*IGAM11+DO1(K,J3,K2,3)*IGAM22)/
                                                    IP3
      GAMMA2 = (QUAL(K,J3,K2,2)*IGAM11+QUAL(K,J3,K2,3)*IGAM2
                                                    2)/IP3
      GO TO 28
   13 IGAM33 = VOL(K,J3,K2,1)
      IF((IGAM11+IGAM22+IGAM33)-IP3)8,15,15
   15 IGAM33 = IGAM11
      IGAM11 = IP3-(IGAM22+IGAM33)
      IK11 = 1
      IK22 = 2
      IK33 = 3
      MIX1 = 2
      GAMMA1 = IP3
      BODMIX = (BOD1(K,J3,K2,1)*IGAM11+BOD1(K,J3,K2,2)*IGAM2
                                                    2+BOD1(K,J3,
     1K2,3)*IGAM33)/IP3
      DOMIX = (DO1(K,J3,K2,1)*IGAM11+DO1(K,J3,K2,2)*IGAM22+D
                                                    O1(K,J3,K2,3
     1)*IGAM33)/IP3
      GAMMA2 = (QUAL(K,J3,K2,1)*IGAM11+QUAL(K,J3,K2,2)*IGAM2
                                                    2+QUAL(K,J3,
     1K2,3)*IGAM33)/IP3
   28 CONTINUE
   30 IP3 = IP4
      RETURN
      END
```

```
      SUBROUTINE TRANSK(K,J1,K1,MIX,IK1,IK2,IK3,IGAM1,IGAM2,
                                                    IGAM3,IQUAN1
     1,JSAVE,KSAVE,E,KEY1)
C     ROUTINE TO TRANSFORM RESERVOIR INTO NEXT PERIOD .
      DIMENSION X(15),T(15),STRDO(15),STRBOD(15),FK(21,3),FK
                                                    M1(21,3),
     1VOL(8,21,3,3),QUAL(8,21,3,3),BOD1(8,21,3,3),DO1(8,21,3
                                                    ,3),
     2SHORTW(15),TEMPAI(15),WIND(15),AIRVAP(15),CC(15),
     3EPIDEP(15),AKA1(15),AK11(12),RDOC(8,3),
     4FLO(15,12),BODIN(12),DOIN(12),VEMEAN(12),DMEAN(12),TRA
                                                    TI(12),TCTRA
     5D(15),ALPHAV(12),ALPHAD(12),BETAV(12),BETAD(12),
     6DISREA(12),TEMIN(12),WATEMP(12),BEGTEM(12),AVGTEM(12),
                                                    AKA2(12)
      COMMON GAMMA1,GAMMA2,VOL,QUAL,DO1,BOD1,FLO,BODIN,DOIN,
                                                    TEMIN,
     1DOMIX,BODMIX,WIND,CC,SHORTW,VEMEAN,DMEAN,TRATI,X,T,TOT
                                                    RAD,
     2ALPHAV,ALPHAD,BETAV,BETAD,DISREA,WATEMP,BEGTEM,AVGTEM,
                                                    AKA2,
     3AK11,STRDO,STRBOD,FACTOR,EPIDEP,AKA1,TEMPAI,AIRVAP
      COMMON NREACH,N,NWL,NL,PMIN,SMIN,PMAX,SMAX,RDOC
      INTEGER GAMMA1,GAMMA2,VOL,QUAL,X,T,SO,SMIN,PMIN,TO,PMA
                                                    X,EPIDEP,
     1SMAX
      JJ = JSAVE
      KK = KSAVE
      IF(NL-1)18,19,18
   19 CALL RESMIX(K,J1,K1,JJ,KK,IQUAN1)
      IF(KEY1.GT.0)  GO TO 21
      ITOVOL = VOL(K+1,J1,K1,1)
      STAGE = DEPTH(ITOVOL)
      WATVAP = WAVP(QUAL(K+1,J1,K1,1))
C     CALCULATE EVAPORATION
      EVAPLO = (73.+7.3*WIND(K))*(WATVAP-AIRVAP(K))*.0005022
                                                    4
      IF(EVAPLO.GT.0.) GO TO 400
      EVAPLO = 0.
  400 STAGE = STAGE-EVAPLO
      ITOVOL = CAPAC(STAGE)
      VOL(K+1,J1,K1,1) = ITOVOL
   21 CONTINUE
      RETURN
C     SUBTRACTING OUTFLOW
   18 IF(MIX-1)1,2,3
    1 VOL(K+1,J1,K1,IK1) = VOL(K,JJ,KK,IK1) - IQUAN1
      IF(IK1-2)4,5,6
    4 VOL(K+1,J1,K1,2) = VOL(K,JJ,KK,2)
      VOL(K+1,J1,K1,3) = VOL(K,JJ,KK,3)
      GO TO 10
    5 VOL(K+1,J1,K1,1) = VOL(K,JJ,KK,1)
      VOL(K+1,J1,K1,3) = VOL(K,JJ,KK,3)
```

```
      GO TO 10
    6 VOL(K+1,J1,K1,1) = VOL(K,JJ,KK,1)
      VOL(K+1,J1,K1,2) = VOL(K,JJ,KK,2)
      GO TO 10
    2 VOL(K+1,J1,K1,IK1) = VOL(K,JJ,KK,IK1) - IGAM1
      VOL(K+1,J1,K1,IK2) = VOL(K,JJ,KK,IK2) - IGAM2
      IF((IK1+IK2)-4)7,8,9
    7 VOL(K+1,J1,K1,3) = VOL(K,JJ,KK,3)
      GO TO 10
    8 VOL(K+1,J1,K1,2) = VOL(K,JJ,KK,2)
      GO TO 10
    9 VOL(K+1,J1,K1,1) = VOL(K,JJ,KK,1)
      GO TO 10
    3 VOL(K+1,J1,K1,IK1) = VOL(K,JJ,KK,IK1) - IGAM1
      VOL(K+1,J1,K1,IK2) = VOL(K,JJ,KK,IK2) - IGAM2
      VOL(K+1,J1,K1,IK3) = VOL(K,JJ,KK,IK3) - IGAM3
C     FIND LAYER FOR INFLOW
   10 IF(T(K)-QUAL(K,JJ,KK,1))11,11,12
C     MIX WITH EPILIMNION
   12 QUAL(K+1,J1,K1,1) = (QUAL(K,JJ,KK,1)*VOL(K+1,J1,K1,1)+
                                                  X(K)*T(K))/
     1(VOL(K+1,J1,K1,1)+X(K))
      VOL(K+1,J1,K1,1) = VOL(K+1,J1,K1,1)+X(K)
      QUAL(K+1,J1,K1,2) = QUAL(K,JJ,KK,2)
      QUAL(K+1,J1,K1,3) = QUAL(K,JJ,KK,3)
      GO TO 20
   11 IF(T(K)-QUAL(K,JJ,KK,3))14,14,15
C     MIX WITH HYPOLIMNION
   14 QUAL(K+1,J1,K1,3) = (QUAL(K,JJ,KK,3)*VOL(K+1,J1,K1,3)+
                                                  X(K)*T(K))/
     1(VOL(K+1,J1,K1,3)+X(K))
      VOL(K+1,J1,K1,3) = VOL(K+1,J1,K1,3)+X(K)
      QUAL(K+1,J1,K1,1) = QUAL(K,JJ,KK,1)
      QUAL(K+1,J1,K1,2) = QUAL(K,JJ,KK,2)
      GO TO 20
C     MIX WITH THERMOCLINE
   15 QUAL(K+1,J1,K1,2) = (QUAL(K,JJ,KK,2)*VOL(K+1,J1,K1,2)+
                                                  X(K)*T(K))/
     1(VOL(K+1,J1,K1,2)+X(K))
      VOL(K+1,J1,K1,2) = VOL(K+1,J1,K1,2)+X(K)
      QUAL(K+1,J1,K1,1) = QUAL(K,JJ,KK,1)
      QUAL(K+1,J1,K1,3) = QUAL(K,JJ,KK,3)
   20 CONTINUE
C     ADDING AND SUBTRACTING INFLOW AND OUTFLOW IS COMPLETED
C     FIND TOTAL VOLUME IN RESERVOIR
      ITOVOL = 0
      DO 30 I=1,NL
   30 ITOVOL = ITOVOL + VOL(K+1,J1,K1,I)
      STAGE = DEPTH(ITOVOL)
      IF(KEY1.GT.0)  GO TO 22
      WATVAP = WAVP(QUAL(K+1,J1,K1,1))
C     CALCULATE EVAPORATION
      EVAPLO = (73.+7.3*WIND(K))*(WATVAP-AIRVAP(K))*.0005022
                                                  4
```

```
      IF(EVAPLO.GT.0.) GO TO 300
      EVAPLO = 0.
  300 STAGE = STAGE-EVAPLO
   22 ITOVOL = CAPAC(STAGE)
C     ADJUST LAYER VOLUMES TO NEW EPILIMNION DEPTH.
      STAG1 = STAGE-EPIDEP(K)
      IVOL5 = CAPAC(STAG1)
      IVOL6 = ITOVOL-IVOL5
      IVOL7 = IVOL6-VOL(K+1,J1,K1,1)
      IF(IVOL7)50,33,33
   33 IF(IVOL7-VOL(K+1,J1,K1,2))34,34,35
   34 QUAL(K+1,J1,K1,1) = (VOL(K+1,J1,K1,1)*QUAL(K+1,J1,K1,1
                                                    )+IVOL7*QUAL
     1(K+1,J1,K1,2))/(VOL(K+1,J1,K1,1)+IVOL7)
      VOL(K+1,J1,K1,1) = VOL(K+1,J1,K1,1)+IVOL7
      VOL(K+1,J1,K1,2) = VOL(K+1,J1,K1,2)-IVOL7
      GO TO 50
   35 IVOL8 = IVOL7-VOL(K+1,J1,K1,2)
      IVOL9 = VOL(K+1,J1,K1,2)
      QUAL(K+1,J1,K1,1) = (VOL(K+1,J1,K1,1)*QUAL(K+1,J1,K1,1
                                                    )+IVOL9*QUAL
     1(K+1,J1,K1,2)+IVOL8*QUAL(K+1,J1,K1,3))/(IVOL7+VOL(K+1,
                                                    J1,K1,1))
      VOL(K+1,J1,K1,1) = VOL(K+1,J1,K1,1)+IVOL7
      VOL(K+1,J1,K1,2) = 0
      VOL(K+1,J1,K1,3) = VOL(K+1,J1,K1,3)-IVOL8
   50 QUAL(K+1,J1,K1,1) = E
C     SET QUALITY FOR NEXT PERIOD.
      DO 60 J=1,NL
      T1 = (QUAL(K+1,J1,K1,J)-32.)*5./9.
      DO1(K+1,J1,K1,J) =(14.652-.41022*T1+.007991*T1*T1-.000
                                                    077774*T1**3
     1)*RDOC(K+1,J)
      BOD1(K+1,J1,K1,J) = 2.0
   60 CONTINUE
      IF(QUAL(K+1,J1,K1,1).GE.QUAL(K+1,J1,K1,2))  GO TO 40
      ITBAR =(VOL(K+1,J1,K1,1)*QUAL(K+1,J1,K1,1)+VOL(K+1,J1,
                                                    K1,2)*QUAL(K
     1+1,J1,K1,2))/(VOL(K+1,J1,K1,1)+VOL(K+1,J1,K1,2))
      QUAL(K+1,J1,K1,1) = ITBAR
      QUAL(K+1,J1,K1,2) = ITBAR
   40 K4 = K+1
      RETURN
      END
      SUBROUTINE  RESMIX(K,J1,K1,JJ,KK,IQUAN1)
C      ROUTINE TO DETERMINE TEMPERATURE OF MIXED RESERVOIR U
                                                    SING
C     EQUATION (18).
      DIMENSION X(15),T(15),STRDO(15),STRBOD(15),FK(21,3),FK
                                                    M1(21,3),
     1VOL(8,21,3,3),QUAL(8,21,3,3),BOD1(8,21,3,3),DO1(8,21,3
                                                    ,3),
     2SHORTW(15),TEMPAI(15),WIND(15),AIRVAP(15),CC(15),
```

```
     3EPIDEP(15),AKA1(15),AK11(12),RDOC(8,3),
     4FLO(15,12),BODIN(12),DOIN(12),VEMEAN(12),DMEAN(12),TRA
                                                  T1(12),TCTRA
     5D(15),ALPHAV(12),ALPHAD(12),BETAV(12),BETAD(12),
     6DISREA(12),TEMIN(12),WATEMP(12),BEGTEM(12),AVGTEM(12),
                                                  AKA2(12)
      COMMON GAMMA1,GAMMA2,VOL,QUAL,DO1,BOD1,FLO,BODIN,DOIN,
                                                  TEMIN,
     1DOMIX,BODMIX,WIND,CC,SHORTW,VEMEAN,DMEAN,TRAT1,X,T,TOT
                                                  RAD,
     2ALPHAV,ALPHAD,BETAV,BETAD,DISREA,WATEMP,BEGTEM,AVGTEM,
                                                  AKA2,
     3AK11,STRDO,STRBOD,FACTOR,EPIDEP,AKA1,TEMPAI,AIRVAP
      COMMON NREACH,N,NWL,NL,PMIN,SMIN,PMAX,SMAX,RDOC
      INTEGER GAMMA1,GAMMA2,VOL,QUAL,X,T,SO,SMIN,PMIN,TO,PMA
                                                  X,EPIDEP,
     1SMAX
      I = 1
      IV1 = VOL(K,JJ,KK,1)
      IV2 = IV1-1QUAN1+X(K)
      IT1 = QUAL(K,JJ,KK,1)
      IT = IT1
      IVBAR = (IV1+IV2)/2
      C = DEPTH(IVBAR)
      AREA = SURAR(D)
      C1 = AREA*30.4/62.4
      C3 = 73.+7.3*WIND(K)
C     BEGIN TRIAL AND ERROR ROUTINE UNTIL AVERAGE TEMPERATUR
                                                  E WILL
C     SATISFY EQUATION (18).
    1 C2 = X(K)*(T(K)-IT1)
      WATVAP = WAVP(IT1)
      AQB = 4.15E-08*.97*(460.+IT1)**4
      AQE = C3*(WATVAP-AIRVAP(K))
      AQH = 0.26*C3*(IT1-TEMPAI(K))
      IDELT = ((TOTRAD(K)-AQE-AQB-AQH)*C1+C2)/IVBAR
      IF(I.GT.20)  CALL EXIT
      ITEST = IT+IDELT/2
      IF(IABS(ITEST-IT1)-1)3,3,2
    2 IF(I.EQ.1)  GO TO 4
      IT1 = ITEST
      IF(I.GT.2)  IT1 = (ITEST+IT2)/2
      IT2 = IT1
      I = I+1
      GO TO 1
    4 IT1 = IDELT/4 +IT
      I = 2
      GO TO 1
    3 QUAL(K+1,J1,K1,1) = IT+IDELT
      VOL(K+1,J1,K1,1) = IV2
      T2 = IT+IDELT
      T1 = (T2-32.)*5./9.
      DO1(K+1,J1,K1,1) =(14.652-.41022*T1+.007991*T1*T1-.C00
                                                  077774*T1**3
```

```
     1)*RDOC(K+1,I)
      BOC1(K+1,JI,K1,1) = 2.0
      RETURN
      END
      SUBROUTINE ALLOWA(K,NW,WINCRE,DMINAL,ALWALO,MNUM,E,AK2
                                                    ,RMEAN,SMEAN
     1,RSTD,SSTD,AK1STD,AK2STD)
C       ROUTINE FOR FINDING ALLOWABLE WASTE LOADING.
      DIMENSION ALA(200)
      DIMENSION X(15),T(15),STRDO(15),STRBOD(15),FK(21,3),FK
                                                    M1(21,3),
     1VOL(8,21,3,3),QUAL(8,21,3,3),BOC1(8,21,3,3),DOI(8,21,3
                                                    ,3),
     2SHORTW(15),TEMPAI(15),WIND(15),AIRVAP(15),CC(15),
     3EPIDEP(15),AKAI(15),AK11(12),RDOC(8,3),
     4FLO(15,12),BODIN(12),DOIN(12),VEMEAN(12),DMEAN(12),TRA
                                                    TI(12),TOTRA
     5D(15),ALPHAV(12),ALPHAD(12),BETAV(12),BETAD(12),
     6DISREA(12),TEMIN(12),WATEMP(12),BEGTEM(12),AVGTEM(12),
                                                    AKA2(12)
      COMMON GAMMAI,GAMMA2,VOL,QUAL,DO1,BOD1,FLO,BODIN,DOIN,
                                                    TEMIN,
     1DOMIX,BODMIX,WIND,CC,SHORTW,VEMEAN,DMEAN,TRATI,X,T,TOT
                                                    RAD,
     2ALPHAV,ALPHAD,BETAV,BETAD,DISREA,WATEMP,BEGTEM,AVGTEM,
                                                    AKA2,
     3AK11,STRDO,STRBOD,FACTOR,EPIDEP,AKA1,TEMPAI,AIRVAP
      COMMON NREACH,N,NWL,NL,PMIN,SMIN,PMAX,SMAX,RDOC
      INTEGER GAMMA1,GAMMA2,VOL,QUAL,X,T,SO,SMIN,PMIN,TO,PMA
                                                    X,EPIDEP,
     ISMAX
      IMAX = 0
      IXX = 0
      IT = 0
      IF(MNUM)1,1,2
    1 IAQ = 0
      K4 = 0
C     FOR FIRST ITERATION SEE IF THE STREAM CAN TAKE ALL WAS
                                                    TE--IF SO
C     NO NEED TO CALC. ALWALO.
    2 I = 1
      BODIN(NW) = 160.*(.02*(TEMIN(NW)-32.)*5./9.+.6)
   50 IF(K.NE.K4) GO TO 51
      IF(GAMMA1.EQ.IAQ) GO TO 60
C     CALL HYDRA FOR DIFFERENT RELEASES.
   51 CALL HYDRA(K)
C     CALL STRMO FOR DIFFERENT RELEASES AND RELEASE TEMPERAT
                                                    URES.
   60 CALL STRMO(E,AK2,K)
C     DETERMINE MINIMUM D.O..
    4 CALL DOMO(DMIN,NW,K,MNUM,RMEAN,SMEAN,RSTD,SSTD,AK1STD,
                                                    AK2STD,IXX,
     1IMAX,KEY)
```

```
      IT = IT+1
      IXX = 1
      TEST = DMIN-DMINAL
      IF(I.GT.1)  GO TO 26
      IF(TEST.GT.0..AND.IT.EQ.1)  GO TO 20
   26 IF(IT.GT.20)  GO TO 24
   27 IF(KEY) 22, 5,23
C     DETERMINE NEW WASTE FOR NEXT TRIAL.
   22 BODIN(NW) = BODIN(NW)-30.
      IF(BODIN(NW).GE.0.)  GO TO 4
      GO TO 10
   23 BODIN(NW) = BODIN(NW)+WINCRE
      GO TO 4
    5 IF(ABS(TEST)-0.02)20,20,7
    7 IF(I.GT.1)  GO TO 18
      ALA(I) = BODIN(NW)
      IF(TEST .LT.0.)  GO TO 15
      BODIN(NW) = BODIN(NW)+WINCRE
   17 DMIN2 = DMIN
      I = I+1
      ALA(I) = BODIN(NW)
      GO TO 4
   15 BODIN(NW) = BODIN(NW)*.75
      GO TO 17
   18 IF(DMIN2.EQ.DMIN)  GO TO 19
      FRAC = (DMIN2-DMINAL)/(DMIN2-DMIN)
      BODIN(NW) = ALA(I-1)-FRAC*(ALA(I-1)-ALA(I))
      GO TO 17
   19 BODIN(NW) = BODIN(NW)+2.*WINCRE
      GO TO 17
   10 ALWALO = 0.
      RETURN
   24 IF(ABS(TEST).GT.0.04) BODIN(NW) = 0.1
C     CALCULATION OF ALLOWABLE LOAD.
   20 ALWALO = FLO(NW,K) *BODIN(NW)*.646*8.34
   21 IAQ = GAMMA1
      K4 = K
      RETURN
      END
      SUBROUTINE HYDRA(K)
C     ROUTINE FOR DETERMINING MEAN VELOCITY,MEAN DEPTH, AND
                                                    TRAVEL TIME.
      DIMENSION X(15),T(15),STRDO(15),STRBOD(15),FK(21,3),FK
                                                    M1(21,3),
     1VOL(8,21,3,3),QUAL(8,21,3,3),BOD1(8,21,3,3),DOI(8,21,3
                                                    ,3),
     2SHORTW(15),TEMPAI(15),WIND(15),AIRVAP(15),CC(15),
     3EPIDEP(15),AKA1(15),AK11(12),RDOC(8,3),
     4FLO(15,12),BODIN(12),DOIN(12),VEMEAN(12),DMEAN(12),TRA
                                                    TI(12),TOTRA
     5D(15),ALPHAV(12),ALPHAD(12),BETAV(12),BETAD(12),
     6DISREA(12),TEMIN(12),WATEMP(12),BEGTEM(12),AVGTEM(12),
                                                    AKA2(12)
```

```
      COMMON GAMMA1,GAMMA2,VOL,QUAL,DC1,BOD1,FLC,BODIN,DOIN,
                                                        TEMIN,
     1DOMIX,BODMIX,WIND,CC,SHORTW,VEMEAN,DMEAN,TRATI,X,T,TOT
                                                        RAD,
     2ALPHAV,ALPHAD,BETAV,BETAD,DISREA,WATEMP,BEGTEM,AVGTEM,
                                                        AKA2,
     3AK11,STRDO,STRBOD,FACTOR,EPIDEP,AKA1,TEMPAI,AIRVAP
      COMMON NREACH,N,NWL,NL,PMIN,SMIN,PMAX,SMAX,RDOC
      INTEGER GAMMA1,GAMMA2,VOL,QUAL,X,T,SO,SMIN,PMIN,TO,PMA
                                                        X,EPIDEP,
     1SMAX
      AQ = 0.
      FLO(1,K) = GAMMA1*FACTOR
      DO 20 I=1,NREACH
      AQ = AQ+FLO(I,K)
      DMEAN(I) = ALPHAD(I)*AQ**BETAD(I)
      VEMEAN(I) = ALPHAV(I)*AQ**BETAV(I)
      TRATI(I) = DISREA(I)/(16.36*VEMEAN(I))
   20 CONTINUE
      RETURN
      END
      SUBROUTINE STRMO(E,AK2,K)
C     ROUTINE FOR DETERMINING STREAM TEMPERATURE,REAERATION
                                                        AND
C     DEOXYGENATION MEANS.
      DIMENSION X(15),T(15),STRDO(15),STRBOD(15),FK(21,3),FK
                                                        M1(21,3),
     1VOL(8,21,3,3),QUAL(8,21,3,3),BOD1(8,21,3,3),DC1(8,21,3
                                                        ,3),
     2SHORTW(15),TEMPAI(15),WIND(15),AIRVAP(15),CC(15),
     3EPIDEP(15),AKA1(15),AK11(12),RDOC(8,3),
     4FLO(15,12),BODIN(12),DOIN(12),VEMEAN(12),DMEAN(12),TRA
                                                        TI(12),TOTRA
     5D(15),ALPHAV(12),ALPHAD(12),BETAV(12),BETAD(12),
     6DISREA(12),TEMIN(12),WATEMP(12),BEGTEM(12),AVGTEM(12),
                                                        AKA2(12)
      COMMON GAMMA1,GAMMA2,VOL,QUAL,DC1,BOD1,FLO,BODIN,DOIN,
                                                        TEMIN,
     1DOMIX,BODMIX,WIND,CC,SHORTW,VEMEAN,DMEAN,TRATI,X,T,TOT
                                                        RAD,
     2ALPHAV,ALPHAD,BETAV,BETAD,DISREA,WATEMP,BEGTEM,AVGTEM,
                                                        AKA2,
     3AK11,STRDO,STRBOD,FACTOR,EPIDEP,AKA1,TEMPAI,AIRVAP
      COMMON NREACH,N,NWL,NL,PMIN,SMIN,PMAX,SMAX,RDOC
      INTEGER GAMMA1,GAMMA2,VOL,QUAL,X,T,SO,SMIN,PMIN,TO,PMA
                                                        X,EPIDEP,
     1SMAX
      TI = 0.
      AQ = 0.
      TEMIN(1) = GAMMA2
      DO 20 I=1,NREACH
      TI = (TEMIN(I)*FLO(I,K)+AQ*TI)/(AQ+FLO(I,K))
      AQ = AQ+FLO(I,K)
```

```
      BEGTEM(I) = TI
      VM1 = VEMEAN(I)*864CC.
      AX = VM1*TRATI(I)
      R2 = AK2*AX/(62.4*DMEAN(I)*VM1)
      TI = (TI-E)*EXP(-R2)+E
      WATEMP(I) = TI
      T1 = (BEGTEM(I)-32.)*5./9.
      T2 = (TI-32.)*5./9.
      AK22 = 5.827*(VEMEAN(I)**.924)/(DMEAN(I)**1.705)
      TT= (T1+T2)/2.-20.
      AK11(I) = AKA1(K)*EXP(TT*.046)
      AKA2(I) = AK22*EXP(TT*.016)
   20 AVGTEM(I) = TT
      RETURN
      END
      SUBROUTINE DOMC(DMIN,NW,K,MNUM,RMEAN,SMEAN,RSTD,SSTD,A
                                                    K1STD,
     1AK2STD,IXX,IMAX,KEY)
C     ROUTINE TO CALCULATE MINIMUM D.C..
      DIMENSION X(15),T(15),STRDO(15),STRBOD(15),FK(21,3),FK
                                                    M1(21,3),
     1VOL(8,21,3,3),QUAL(8,21,3,3),BOD1(8,21,3,3),DO1(8,21,3
                                                    ,3),
     2SHORTW(15),TEMPAI(15),WIND(15),AIRVAP(15),CC(15),
     3EPIDEP(15),AKA1(15),AK11(12),RDCC(8,3),
     4FLO(15,12),BODIN(12),DOIN(12),VEMEAN(12),DMEAN(12),TRA
                                                    TI(12),TOTRA
     5D(15),ALPHAV(12),ALPHAD(12),BETAV(12),BETAD(12),
     6DISREA(12),TEMIN(12),WATEMP(12),BEGTEM(12),AVGTEM(12),
                                                    AKA2(12)
      COMMON GAMMA1,GAMMA2,VOL,QUAL,DO1,BOD1,FLO,BODIN,DOIN,
                                                    TEMIN,
     1DOMIX,BODMIX,WIND,CC,SHORTW,VEMEAN,DMEAN,TRATI,X,T,TOT
                                                    RAD,
     2ALPHAV,ALPHAD,BETAV,BETAD,DISREA,WATEMP,BEGTEM,AVGTEM,
                                                    AKA2,
     3AK11,STRDO,STRBOD,FACTOR,EPIDEP,AKA1,TEMPAI,AIRVAP
      COMMON NREACH,N,NWL,NL,PMIN,SMIN,PMAX,SMAX,RDCC
      INTEGER GAMMA1,GAMMA2,VOL,QUAL,X,T,SO,SMIN,PMIN,TO,PMA
                                                    X,EPIDEP,
     1SMAX
      DIMENSION FLOW(16),DISOXY(15),ALA(15),ALAD(15),DMINM(1
                                                    5)
      KEY = 0
      DMIN = 20.
      IF(IXX.EQ.0)  GO TO 1
      I1 = NW
      GO TO 2
    1 BEGTEM(1) = GAMMA2
      DOIN(1) = DOMIX
      BODIN(1) = BODMIX
      ALA(1) = 0.
      FLOW(1) = 0.
```

```
      DISOXY(1) = C.
      I1 = 1
C     BEGIN CALCULATION OF D.C. AT END OF EACH REACH.
    2 DO 20 I=I1,NREACH
      AQ = FLOW(I)+FLO(I,K)
      ALAO(I) = (BODIN(I)*FLO(I,K)+FLOW(I)*ALA(I))/AQ
      IF(I.EQ.NW) VL = (.05*FLO(I,K)*BODIN(NW)+FLOW(I)*VARLL
                                                    )/AQ
      ALII = ALAO(I)*EXP(.02*AVGTEM(I))
      DOI = (DOIN(I)*FLO(I,K)+FLOW(I)*DISOXY(I))/AQ
      FLOW(I+1) = AQ
      T2 = 5.*(BEGTEM(I)-32.)/9.
       CSI = 14.652-.41022*T2+.007991*T2*T2-.00077774*T2**3
                                                    .
       DAI = CSI-DOI
      T2 = 5.*(WATEMP(I)-32.)/9.
       CSF = 14.652-.41022*T2+.007991*T2*T2-.00077774*T2**3
                                                    .
      B = (CSF-CSI)/TRATI(I)
      B1 = AK11(I)*TRATI(I)
      B2 = AKA2(I)*TRATI(I)
      B3 = EXP(-B1)
      B4 = EXP(-B2)
      IYY = 0
C     USE STOCHASTIC FORMULATION.
      CALL DODEF(B,B3,B4,AK11(I),AKA2(I),ALII,DAI,DAF,RMEAN,
                                                    SMEAN,RSTD,
     1SSTD,AK1STD,AK2STD,ALLA,IYY,IMAX,I,DAB,TRATI(I),VL,VAR
                                                    LL)
      ALA(I+1)=ALLA/EXP(.02*AVGTEM(I))
      DISOXY(I+1)=CSF-DAF
      DMINM(I) = CSF-DAB
      IF(DISOXY(I+1).LT.0.)   DISOXY(I+1) = 0.
      IF(DMINM(I).LT.0.)  DMINM(I) = C.
      IF(I.LE.NW) GO TO 20
      IF(DISOXY(I).LE.DISOXY(I+1).AND.DISOXY(I).LT.DISOXY(I-
                                                    1)) GO TO 3
      GO TO 20
C     WHEN D.C. CURVE BEGINS TO INCREASE, CALCULATE MINIMUM
                                                    BY EXAMINING
C     THE D.O. AT THE THIRD POINTS OF THE LAST TWO REACHES.
    3 IF(BODIN(NW).EQ.0.) GO TO 19
      J = I-1
      DO 10 I2=1,2
      TT = TRATI(J)/3.
      ALII = ALAO(J)*EXP(.02*AVGTEM(J))
      DOI = (DOIN(J)*FLO(J,K)+FLOW(J)*DISOXY(J))/(FLOW(J)+FL
                                                    O(J,K))
      T2 = 5.*(BEGTEM(J)-32.)/9.
       CSI = 14.652-.41022*T2+.007991*T2*T2-.00077774*T2**3
                                                    .
       DAI = CSI-DOI
      T2 = 5.*(WATEMP(J)-32.)/9.
```

```
      CSF = 14.652-.41022*T2+.007991*T2*T2-.00077774*T2**3
      B = (CSF-CSI)/IRATI(J)
      DO 15 J1=1,2
      B1 = AK11(J)*TT
      B2 = AKA2(J)*TT
      B3 = EXP(-B1)
      B4 = EXP(-B2)
      IYY = 1
      CALL DODEF(B,B3,B4,AK11(J),AKA2(J),ALII,DAI,DAF,RMEAN,
                                                  SMEAN,RSTD,
     1SSTD,AK1STD,AK2STD,ALLA,IYY,IMAX,I,DAB,TT,VL,VARLL)
      DISO = (CSI+B*TT)-DAB
      IF(DISO.LT.0.) DISO = 0.
      IF(DMIN.LT.DMINM(J-1).AND.DMIN.LT.DISO)  GO TO 15
      IF(DISO.LT.DMINM(J-1))  DMIN = DISO
      IF(DMINM(J-1).LT.DISO) DMIN = DMINM(J-1)
   15 TT = 2.*TT
      IF(I2.GT.1) RETURN
      IF(DMIN.LT.DMINM(J))  RETURN
      DMIN = DMINM(J)
   10 J = J+1
   19 IF(DMIN-DMINM(I))20,20,6
    6 DMIN = DMINM(I)
   20 CONTINUE
      IF(DMIN.NE.20.) RETURN
      SLOPE = DISOXY(11)-DISOXY(10)
      IF(SLOPE.EQ.0.) SLOPE = DISOXY(10)-DISOXY(9)
      IF(SLOPE.EQ.0.) SLOPE = DISOXY(9)-DISOXY(8)
      IF(SLOPE.GT.0.)  KEY = 1
C     IF KEY IS POSITIVE, THEN NO OXYGEN SAG OCCURRED.
      IF(SLOPE.LT.0.)  KEY =-1
C     IF KEY IS NEGATIVE, THEN THE STREAM NEVER BEGAN TO REC
                                                  OVER AND THE
C     LOAD MUST BE DECREASED.
      DMIN = DMINM(NREACH)
      RETURN
      END
      SUBROUTINE DODEF(B,P,C,K1,K2,LA,DA,DAF,R,S,RS,SS,K1S,K
                                                  2S,ALLA,IYY,
     1IMAX,I,DAB,T,VL,VARLL)
C     STOCHASTIC DISSOLVED OXYGEN EQUATION.
      REAL K1,K2,LA,K1S,K2S,M,L,N,K12,K22,M2,M3
      DIMENSION C1(13),C2(13),C3(13),C4(13),C5(13),C6(13),C7
                                                  (13),C8(13),
     1C9(13),C10(13),C11(13),C12(13),C14(13),VK1(13),VK2(13)
                                                  ,C13(13),
     2C15(13),C16(13)
      DIMENSION VARL(14),VARD(14),AL(14),AD(14)
      J = I
      K = I
      IF(I.EQ.4)  VARL(I) = VL
      VARL(1) = 0.
```

```
      VARD(1) = C.
      AL(K) = LA
      AD(K) = DA
      IF(IYY.EQ.1)  J = 13
      IF(J-IMAX) 2,2,1
    1 N = P-Q
      K12 = K1*K1
      K22 = K2*K2
      M = K2-K1
      M2 = M*M
      SK1 = K1*K1S
      SK2 = K2*K2S
      VK1(J) = SK1*SK1
      VK2(J) = SK2*SK2
      Y = B+R+S
      T1 = T*P
      T2 = T*Q
      P1 = 1.-P
      Q1 = 1.-Q
      C6(J) = Y*(T2-Q1/K2)/K2
      C8(J) = T1*T
      C9(J) = Y*(T2*(-2./K2-T)+2.*Q1/K22)/K2
      C10(J) = -N/M+Q1/K2
      C11(J) = Q1/K2
      C12(J) = K1*N/M
      C13(J) = R*(T1-P1/K1)/K1
      C14(J) = R*(2.*P1/K12-T1*(T+2./K1))/K1
      C15(J) = Y*Q1/K2
      C16(J) = N/M
      IF(ABS(M).LT.0.01)  GO TO 3
      C1(J) = (K2*N/M-K1*T1)/M
      C2(J) = R*(T1-N/M)/M
      C5(J) = (T2-N/M)/M
      GO TO 4
    3 C1(J) = T*P*(1.-K1*T)
      C2(J) = R*T*T*P
      C5(J) = -T*T*Q
    4 IF(ABS(M).LT.0.02) GO TO 5
      C3(J) = K2*(-T1+2.*N/M)/M2-T1*(1.-K1*T+K1/M)/M
      C4(J) = R*(T1*(2./M-1)-2.*N/M2)/M
      C7(J) =(T2*(-T-2./M)+2.*N/M2)/M
      GO TO 2
    5 C3(J) = P*(T*T*(K1*T-2.))
      C4(J) = C2(J)*T
      C7(J) = T*T*T*Q
    2 C17 = K1*AL(K)-R
C     EVALUATION OF PARTIAL DERIVATIVES AS GIVEN IN APPENDIX
                                                          A.
      PDPK1 = ((AL(K)*C1(J)+C2(J))**2)*VK1(J)
      PDPK12 = (AL(K)*C3(J)+C4(J))*VK1(J)
      PDPK2 = (((C17)*C5(J)-AD(K)*T*C+C6(J))**2)*VK2(J)
      PDPK22 = (C17*C7(J)+AD(K)*C8(J)+C9(J))*VK2(J)
      PDPR = C10(J)*C10(J)*RS*RS
```

```
      PDPS = C11(J)*C11(J)*SS*SS
      PDPLA = C12(J)*C12(J)*VARL(K)
      PDPDA = Q*Q*VARD(K)
      PLPK1 = ((-T1*AL(K)+C13(J))**2)*VK1(J)
      PLPK12 =(T*T*P*AL(K)+C14(J))*VK1(J)
      PLPR = ((P1/K1)**2)*RS*RS
      PLPLA = P*P*VARL(K)
      D = C17*C16(J)+AD(K)*Q+C15(J)
      EPD = D+.5*(PDPK12+PDPK22)
      VARD(K+1) = PDPK1+PDPK2+PDPR+PDPS+PDPDA+PDPLA
      EPL = (AL(K)-R/K1)*P+R/K1+.5*PLPK12
      VARL(K+1) = PLPK1+PLPR+PLPLA
      DAF = EPD
      DAB = EPD+1.645*SQRT(VARD(K+1))
      ALLA = EPL
      IMAX = K
      IF(I.EQ.3)  VARLL = VARL(K+1)
      RETURN
      END
      FUNCTION DEPTH(ITOV)
C     STAGE VS. VOLUME FUNCTION.
      TOV = ITOV
      R = 76.25396825
      S = (4.18062-TOV/1000000.)/.00063
      TEMP = R*R-S
      IF(TEMP.LT.0.)  TEMP = 0.
      DEPTH = 350.-(R-SQRT(TEMP))
      RETURN
      END
      FUNCTION SURAR(FOOT)
C     SURFACE AREA VS. STAGE FUNCTION.
      D = 350.-FOOT
      SURAR = (104.66165-1.92119*D+.0095*D*D)*1000.
      RETURN
      END
      FUNCTION CAPAC(FOOT)
C     VOLUME VS. STAGE FUNCTION.
      D = 350.-FOOT
      CAPAC = (4.18062-.09608*D+.00063*D*D)*1000000.
      RETURN
      END
      FUNCTION WAVP(ITEMP)
C     WATER VAPOR PRESSURE BY FIFTH ORDER  LAGRANGIAN POLYNO
                                                    MIAL.
      X = ITEMP
      A1 = X-40.
      A2 = X-63.3
      A3 = X-76.
      A4 = X-88.
      A5 = X-100.
      C1 = A2*A3*A4*A5*6.2/2415744.
      C2 = A1*A3*A4*A5*15./(-268239.4559)
      C3 = A1*A2*A4*A5*23./131673.6
```

```
      C4 = A1*A2*A3*A5*33.8/(-170726.4)
      C5 = A1*A2*A3*A4*48.8/634176.
      WAVP = C1+C2+C3+C4+C5
      RETURN
      END
      FUNCTION CLODCO(VAPPRE,CCOVER)
C     FUNCTION PROGRAM TO CALCULATE ATMOSPHERIC RADIATION FA
                                                        CTOR
C     VAPPRE = VAPOR PRESSURE OF AIR IN MM. OF MERCURY
C     CCOVER = CLOUD COVER IN TENTHS
      NCOVER = 1+CCOVER
      GO TO (1,2,3,4,5,6,7,8,9,10,11),NCOVER
    1 CLODCO = .738 + .154*VAPPRE/25.4
      RETURN
    2 CLODCO = .75 + .15*VAPPRE/25.4
      RETURN
    3 CLODCO = .759 + .148*VAPPRE/25.4
      RETURN
    4 CLODCO = .772 + .143*VAPPRE/25.4
      RETURN
    5 CLODCO = .783 + .138*VAPPRE/25.4
      RETURN
    6 CLODCO = .792 + .135*VAPPRE/25.4
      RETURN
    7 CLODCO = .80 + .133*VAPPRE/25.4
      RETURN
    8 CLODCO = .811 + .130*VAPPRE/25.4
      RETURN
    9 CLODCO = .825 + .118*VAPPRE/25.4
      RETURN
   10 CLODCO = .845 + .105*VAPPRE/25.4
      RETURN
   11 CLODCO = .865 + .090*VAPPRE/25.4
      RETURN
      END
```

C. GLOSSARY OF PROGRAMMING TERMS

AIRVAP: Air-vapor pressure.

AKA1: Deoxygenation coefficient at 20°C.

AKA2: Reaeration coefficient adjusted to temperature.

AK11: Deoxygenation coefficient adjusted to temperature.

AK1STD: Constant for computing variance of deoxygenation coefficient.

AK2SD: Constant for computing variance of reaeration coefficient.

ALLOWA: Subroutine for calculating allowable waste load.

ALPHAD: Constant in Equation (67).

ALPHAV: Constant in Equation (66).

AUXCOS: Subroutine for calculating cost of auxiliary waste treatment unit.

AVGTEM: Average temperature difference above or below 20°C in reach.

BEGTEM: Temperature at beginning of reach.

BETAD: Exponent in Equation (67).

BETAV: Exponent in Equation (66).

BOD1: Matrix for BOD in the reservoir.

BODI: Initial BOD in reservoir.

BODIN: BOD of tributary flow entering reach.

BODMIX: BOD in reservoir release.

CAPAC: Function for calculating reservoir storage.

CC: Cloud cover.

CLODCO: Function for calculating β_o in Equation (2).

DEC1: Matrix for decisions on quantity to be released.

DEC2: Matrix for decisions on temperature of release water.

DEC3: Matrix for decisions on release from layer one.

DEC4: Matrix for decisions on release from layer two.

DEC5: Matrix for decisions on release from layer three.

DEC6: Matrix for decisions on quantity released from layer one.

DEC7: Matrix for decisions on quantity released from layer two.

DEC8: Matrix for decisions on quantity released from layer three.

DEC9: Matrix for decisions on quantity released in previous period.

DEC10: Matrix for decisions on released layer in previous period.

DEPTH: Function for calculating depth of reservoir storage.

DISCHA: Subroutine for determining quality of reservoir release.

DISREA: Distance of reach.

DMEAN: Mean depth in reach.

DMINAL: Minimum allowable DO.

DO1: Matrix for DO in reservoir.

DODEF: Subroutine for calculating DO deficit.

DOI: Initial DO in reservoir.

DOIN: DO in tributary flow to reach.

DOMIX: DO in reservoir release.

DOMO: Subroutine for determining minimum DO.

DORELS: DO level to be maintained by aeration at reservoir.

EPIDEP: Depth of epilimnion.

EQUIL: Subroutine for calculating equilibrium temperature.

FACTOR: Constant to convert cfs to acre-ft.

FK: Matrix of costs for present period.

FKM1: Matrix of costs in previous period.

FLO: Matrix of tributary flows.

GAMMA1: Release quantity from reservoir.

GAMMA2: Release temperature from reservoir.

HYDRA: Subroutine for calculating mean depth, mean velocity and travel time in reaches.

IAERAT: Control variable -- positive means aerate release.

IK8: Control variable -- positive means release from hypolimnion.

INCRE: Increment for discharge rates.

KEY1: Control variable -- positive means do not calculate evaporation loss.

N: Number of time intervals.

NL: Number of layers.

NREACH: Number of reaches.

NW: Reach into which waste is discharged.

NWL: Number of withdrawal rates.

PMAX: Maximum withdrawal rate.

PMIN: Minimum withdrawal rate.

QUAL: Matrix for temperatures in reservoir.

RADCAL: Subroutine for calculating TOTRAD, PMIN, PMAX and PMIN.

RDOC: Reservoir dissolved oxygen constants.

RESMIX: Subroutine for calculating temperature of mixed reservoir.

RMEAN: Composite term, r, in Equation (52).

RSTD: Standard deviation of RMEAN.

SO: Initial storage in reservoir.

SHORTW: Short-wave solar radiation.

SMAX: Maximum allowable storage.

SMEAN: Composite term, s, in Equation (53).

SMIN: Minimum allowable storage.

SSTD: Standard deviation of SMEAN.

STRBOD: Inflowing BOD to reservoir.

STRDO: Inflowing DO to reservoir.

STRMO: Subroutine for calculating stream temperatures.

SURAR: Function for calculating reservoir surface area.

T: Temperature of inflows to reservoir.

TO: Initial temperature in reservoir.

TEMIN: Temperature of tributary flows to reaches.

TEMPAI: Air temperature.

TOTRAD: Total absorbed radiation.

TRATI: Travel time in reach.

TRANSK: Subroutine to transfer reservoir state variables according to optimum decisions.

VEMEAN: Mean velocity in reach.

VOL: Matrix for volumes in various layer of reservoir.

WATEMP: Water temperature at end of reach.

WAVP: Function for calculating vapor pressure at temperature of surface water.

WIND: Wind speed.

WINCRE: Increment of waste to be used in determining allowable load.

X: Inflow volumes to reservoir.

REFERENCES

1. "National Water Pollution Control Program," Jour. AWWA, 41, 40 (1949).

2. U. S. Congress, Federal Water Pollution Control Act Amendments of 1961, Public Law 88, 87th Cong. (1961).

3. Water Quality Act of 1965, 33, U.S.C.A., 466 (Supp. 1966).

4. "Guidelines for Establishing Water Quality Standards for Interstate Waters," U. S. Dept. of Interior, Federal Water Pollution Control Administration (May, 1966).

5. Davis, R. K., "Some Aspects of Advanced Waste Treatment," Jour. WPCF, 37, 2, 1617 (1965).

6. "Report of the Committee on Water Quality Criteria," U. S. Dept. of Interior, Federal Water Pollution Control Administration (April, 1968).

7. Bramhall, D. F. and E. S. Mills, "Alternative Methods of Improving Stream Quality: An Economic and Policy Analysis," Water Resources Research, 2, 3, 355 (1966).

8. Young, G. D., "Reservoir Management: The Tradeoff Between Low Flow Regulation and Flood Control," Water Resources Research, 4, 3, 507 (1968).

9. Loucks, D. P., "Risk Evaluation in Sewage Treatment Plant Design," Proc. ASCE, Jour. San. Eng. Div., 93, 1, 25 (1967).

10. Moreau, D. H., "Mathematical Models for Industrial Waste Disposal Systems," Harvard Water Resources Group, Harvard University, Cambridge (1967).

11. Ward, J. C., "Annual Variation of Stream Water Temperature," Proc. ASCE, Jour. San. Eng. Div., 89, 6, 1 (1963).

12. Churchill, M. A., "Effects of Storage on Impoundments on Water Quality," Paper No. 2928, Transactions ASCE, 123, 419 (1958).

13. Churchill, M. A., "Control of Temperature Through Streamflow Regulation," Fnvironmental Health Series, WSPC, Symposium on Streamflow Regulation for Quality Control, U. S. Department of Health, Education, and Welfare, Public Health Service, Cincinnati, Ohio (June, 1965).

14. Edinger, J. E. and J. C. Geyer, "Heat Exchange in the Environment," Publication 65-902, Edison Electric Institute, New York 17, New York (1965).

15. Johns Hopkins University, "Heat Dissipation in Flowing Streams," Report of the 1960-61 Advanced Seminar, Department of Sanitary Engineering and Water Resources (December, 1961).

16. Birge, E. A. and C. Juday, "Transmission of Solar Radiation by the Waters of Inland Lakes," Transactions of the Wisconsin Academy of Sciences, 24, 508 (1929).

17. Raphael, J. M., "Predictions of Temperatures in Rivers and Reservoirs," Proc. ASCE Jour. Power Div., 88, 2, 157 (1962).

18. Bowen, I. S., "The Ratio of Heat Losses by Conduction and by Evaporation from any Water Surface," Physical Review, 27, 2 (1906).

19. Wunderlich, W. O. and R. A. Elder, "The Mechanics of Stratified Flow in Reservoirs," Proceedings, Reservoir Fishery Resources Symposium, American Fisheries Society, Athens, Georgia (April, 1967).

20. Orlob, G. T., "A Mathematical Model of Thermal Stratification in Deep Reservoirs," Presented at the Annual Meeting of the American Fisheries Society at Portland, Oregon (September 24, 1965).

21. Hutchinson, G. E., "A Treatise on Limnology, Vol. 1, John Wiley and Sons, Inc., New York (1957).

22. Dake, J. M. K. and D. R. F. Harleman, "An Analytical and Experimental Investigation of Thermal Stratification in Ponds and Lakes," Technical Report No. 99, M.I.T. Hydrodynamics Laboratory, Cambridge (1966).

23. Churchill, M. A., "Effects of Density Currents in Reservoirs on Water Quality," Water and Sewage Works - Reference Number - 1965.

24. Koberg, G. E. and M. E. Ford, "Elimination of Thermal Stratification in Reservoirs and the Resulting Benefits," USGS Water Supply Paper 1809-M (1965).

25. Delay, W. H. and J. Seaders, "Predicting Temperatures in River and Reservoirs," Proc. ASCE, Jour. San. Eng. Div., 92, 1, 115 (1966).

26. Orlob, G. R. and L. G. Selna, "Mathematical Simulation of Thermal Stratification in Deep Reservoirs," A.S.C.E. Specialty Conference on Current Research into the Effects of Reservoirs on Water Quality, Portland, Oregon (January, 1968).

27. Nicolson, G. S., "Discussion: Mathematical Simulation of Thermal Stratification in Deep Reservoirs," A.S.C.E. Specialty Conference on Current Research into the Effects of Reservoirs on Water Quality, Portland, Oregon (January, 1968).

28. Huber, W. C. and D. R. F. Harleman, "Laboratory and Analytical Studies of the Thermal Structure of Reservoirs," Technical Report No. 112, M.I.T. Hydrodynamics Laboratory, Cambridge (1968).

29. Koh, R. C. Y., Viscous Stritified Flow Towards a Line Sink," W. M. Keck Laboratory Report KH-R-6, California Institute of Technology (1964).

30. Kao, T. W., "The Phenomenon of Blocking in Stratified Flows," Ph.D. Dissertation, University of Michigan, Ann Arbor (1963).

31. Ross, R. L. and N. J. MacDonald, "The Effect of Libby Reservoir, Montana on Water Temperatures," ASCE Hydraulics Division 16th Annual Specialty Conference, "Computer Applications in Hydraulic and Water Resources Engineering," M.I.T., Cambridge (August, 1968).

32. Posey, F. H. and J. W. Dewitt, "Effects of Reservoir Impoundment on Water Quality," Presented at the A.S.C.E. National Meeting on Environmental Engineering, Chattanooga, Tennessee (May, 1968).

33. "The Prediction of Stream Reaeration Rates," Tennessee Valley Authority, Division of Health and Safety, Environmental Hygiene Branch, Chattanooga, Tennessee (1962).

34. Babbitt, H. E. and E. R. Baumann, Sewerage and Sewage Treatment, John Wiley and Sons, New York (1958).

35. Churchill, M. A. and W. R. Nicholas, "Effects of Impoundments on Water Quality," Proc. ASCE, Jour. San. Eng. Div., 93, 6, 73 (1967).

36. Kittrell, F. W., "Effects of Impoundments on Dissolved Oxygen Resources," Sewage and Industrial Wastes, 31, 9, 1065 (1959).

37. Thackston, E. L. and R. E. Spuce, "Review of Supplemental Reaeration of Flowing Streams," Jour. WPCF, 38, 10, 1614 (1966).

38. Wiley, A. J., Lueck, B. F., Scott, R. H., and T. F. Wisniewski, "Commercial Scale Stream Reaeration", Jour. WPCF, 34, 4, 401 (1962).

39. Symons, J. M., W. H. Irwin, and G. G. robeck, "Impoundment Water Changes Caused by Mxing,"Proc. ASCE, J. San, Eng. Div., 93, 2, 181 (1967).

40. Irwin, W. H., J. M. Symons and G. G. Robeck, "Impoundment Destratification by Mechanical Pumping," Proc. ASCE, J. San Eng. Div., 92, 6, 21 (1966).

41. Worley, J. L., F. J. Burgess, and W. W. Towne, "Identification of Low Flow Augmentation Requirements for Water Quality Control by Computer Techniques," Jour. WPCF, 37, 5, 659 (1965).

42. Feigner, K. D., "An Evaluation of Temperature Reduction on Low Flow Augmentation Requirements for Dissolved Oxygen Control," Thesis, Oregon State University, Corvallis, Oregon (1963).

43. Duttweiler, D. W., "A Mathematical Model of Stream Temperature," Ph.D. Dissertation, The Johns Hopkins University, Baltimore, Maryland (1963).

44. Velz, C. J. and J. J. Gannon, "Forecasting Heat Loss in Ponds and Streams," Jour. WPCF, 34, 4, 392 (1960).

45. Streeter, H. W. and E. B. Phelps, "A Study of the Pollution and Natural Prufication of the Ohio River," United States Public Health Service, Public Health Bulletin No. 146 (February, 1925).

46. Langbein, W. B. and W. H. Durum, "The Reaeration Capacity of Streams," Geological Survey Circular 542, U.S.G.S. Washington, D. C. (1967).

47. Churchill, M. A. and R. A. Buckingham, "Statistical Method for Analysis of Stream Purification Capacity," Sewage and Industrial Wastes, 28, 4, 517 (1956).

48. Kothandaraman, Veerasamy, "Probabilistic Analysis of Wastewater Treatment and Disposal Systems," University of Illinois, Water Resources Center, Research Report No. 14, Urbana (1968).

49. Streeter, H. W., Wright, C. T. and R. W. Kehr, "Measures of Natural Oxidation in Polluted Streams, III, An Experimental Study of Atmospheric Reaeration Under Stream Flow Conditions," Sewage Works Journal, 8, 2, 282 (March, 1936).

50. Krenkel, P. A. and G. T. Orlob, "Turbulent Diffusion and the Reaeration Coefficient," Proc. ASCE, Jour. San. Eng. Div., 88, 2, 53 (1962).

51. Thomas, H. A., "Pollution Load Capacities of Streams," Water and Sewage Works, 9, 3, 425 (1948).

52. Camp, T. R., "Field Estimates of Oxygen Balance Parameters," Proc. ASCE, Jour. San. Eng. Div., 91, 5, 1 (1965).

53. Moreau, D. H. and E. E. Pyatt, "Uncertainty and Data Requirements in Water Quality Forecasting: A Simulation Study," Final Report to U. S. Geological Survey, University of Florida, Gainesville (1968).

54. Liebman, J. C., "The Optimal Allocation of Stream Dissolved Oxygen Resources," Cornell University Water Resources Center, Ithaca, New York (1965).

55. Meyer, P. L., Introductory Probability and Statistical Applications, Addison-Wesley, Reading, Massachusetts (1965).

56. Fair, G. M., and J. C. Geyer, Water Supply and Waste-Water Disposal, John Wiley and Sons, Inc., New York (1965).

57. Krenkel, P. A., W. A. Cawley, and V. A. Minch, "The Effects of Impoundments on River Waste Assimilative Capacity," Jour. WPCF, 37, 9, 1203 (1965).

58. Kerri, K. D., "An Economic Approach to Water Quality Control," Jour. WPCF, 38, 12, 1883 (1966).

59. ReVelle, C. S., Loucks, D. P. and W. R. Lynn, "Linear Programming Applied to Water Quality Management," Water Resources Research, 4, 1, 1 (1968).

60. Little, J. D. C., "The Use of Storage Water in a Hydroelectric System," J. Operations Research Society of America, 3, 187 (1955).

61. Young, K. G., "Techniques for Finding Reservoir Operating Rules," Harvard Water Resources Group, Harvard University, Cambridge (1966).

62. Bellman, R. E., Dynamic Programming, Princeton University Press, Princeton, New Jersey (1957).

63. Bellman, R. E. and S. E. Dreyfus, Applied Dynamic Programming, Princeton University Press, Princeton, New Jersey (1962).

64 "Savannah Basin," Plan for the Development of the Land and Water Resources of the Southeast River Basins, Appendix 1, United States Study Commission, Southeast River Basins (1963).

65. "Rules of the Georgia Water Quality Control Board," 47 Trinity Avenue, S. W., Atlanta, Georgia.

66. "Appendix A - Clark Hill Reservoir," Savannah River Basin, Georgia and South Carolina, U. S. Army Corps of Engineers, Savannah District Office, Savannah, Georgia.

67. "Savannah Basin," U. S. Study Commission, Southeast River Basins (1961).

68. Climatological Data -- National Summary, U. S. Weather Bureau.

69. Personal communication with Dr. D. P. Loucks.

70. Smith, R., "Cost of Conventional and Advanced Treatment of Wastewater," Jour. NPCF, 40, 9, 1546 (1968).

BIOGRAPHICAL SKETCH

Gilbert S. Nicolson, Jr. was born December 7, 1941, at Birmingham, Alabama. He was graduated from Landon High School, Jacksonville, Florida, in June 1960. He was graduated from the University of Florida in April 1965 with a Bachelor of Civil Engineering. He was commissioned as a Second Lieutenant in the U. S. Army in April 1965 and was granted a delay of active duty for graduate studies. In April 1966, he received a Master of Engineering specializing in Sanitary Engineering from the University of Florida. From May 1966 until the present time he has pursued his work toward the degree Doctor of Philosophy in the Department of Environmental Engineering. From September 1967 until December 1968, he was a Graduate Associate with the U. S. Army Corps of Engineers.

Gilbert S. Nicolson is married to the former Rachael Anne Koontz and is the father of one son. He is a member of Sigma Tau and Phi Kappa Phi. Upon the completion of his studies, he will enter the U. S. Army Medical Service Corps as a Captain.

This thesis was prepared under the direct[illegible] of the chairman of the candidate's supervisory committee and has [illegible]n approved by all members of that committee. It was submitted t[illegible]he Dean of the College of Engineering and to the Graduate Council, and [illegible]as approved as partial fulfillment of the requirements for the degree [illegible]f Doctor of Philosophy.

June, 1969

Robert E Uhrig
Dean, College of Engineering

Dean, Graduate School

Supervisory Committee:

E. E. Pyatt
Chairman

Wayne C. Huber

N. S. Key

A. L. Patterson

6905